

Thermoeconomics

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*Just as the constant increase of entropy is the
basis law of the universe, so it is basic law of
life to be ever more highly structured and to
struggle against entropy*

Vaclav Havel

Preface

The increasing demand for power and material product, driving by the increase of population and a rising standard of live, on one hand and the adverse impact of the current energy conversion technologies, due to emissions, waste disposal and the sign of global earth warning, on the other hand, has been created an energy crisis that is going to decades to come. The long- lasting energy crisis result from the absence of alternative energy resources and conversion technologies that are both friendly to the environment and economically competitive to the present ones. Until new competitive or emerging technologies become available, the cost-effective increase of the conversion efficiencies of current technologies is the only option to reduce the impact of the crisis.

The traditional approaches to the synthesis and the design of energy-intensive systems relied in the past on the intuition of experienced engineers and designers. A low concern was given to fuel consumption and no concern was given to the environment. Today both concerns are increased, this situation did rise to a global level and did pose a hard challenge to the designer and the operators of energy intensive systems. Cost-effective fuel saving became a focus of attention in the design and operation of these systems. The design aspects became a complex multidisciplinary process requiring specialized knowledge in each discipline. The operation aspect became more responsive to any mismanagement of energy emission and waste disposal.

This book provides a rigorous introduction to thermoeconomic analysis, a group of methodologies that study the relationship between cost and efficiency of the energy conversion technologies and which main objective is to lay the foundations of a science of energy resource saving.

The book is intended to use in a course for the doctoral program in *renewable energy and energy efficiency* at the university of Zaragoza, called *Thermoeconomics* with a duration of forty hours.

Our development is aimed at imparting an understanding on the process of cost formation of the energy conversion systems and its mathematical basis, with a major focus on its main applications: cost accounting, optimal design and malfunction diagnosis.

Thus, Chapter 1 presents an overview of the basic concepts of Thermoeconomics, explaining the relationships between the Second Law of Thermodynamics and Economy trough the concept of cost. An historical review of more than a 20 year development is used to explain the main subjects on Thermoeconomics.

Chapter 2 presents the Exergy Cost Theory a methodology that combines the general theory of systems, the exergy concept of thermodynamics and economic principles of cost accounting. This methodology identify the real cost sources at

component level: the capital, operation, and maintenance cost, the cost of energy resources and the cost of exergy losses or irreversibilities. The numerical methods for calculating the exergy and thermoeconomic costs are explained in detail.

The topics initiated in Chapter 2 continues in Chapter 3 with the development of the *Symbolic Exergoeconomics* based on the *Fuel-Product* model. It is a powerful and a systematic approach that studies in-depth the process of cost formation and provide the tools to obtain the mathematical relationship between cost, efficiency and irreversibility. The cost of residues and its impact in the cost of the product is also dealt.

Chapters 4 and 5 deal with the applications of thermoeconomics, based on the results obtained in previous chapter. The design and optimization of thermal systems is discussed in Chapter 4. The development is based on decomposition strategies that consider the optimization problem of a system as a set of sub-optimization problems. The energy audit and thermoeconomic diagnosis is dealt in Chapter 5. The principle non-equivalence of irreversibilities and the fuel impact formula, introduced in chapter 3, are used to identify the malfunction of the individual components and the dysfunctions that produces in the whole system.

To end the main matter of the book Chapter 6 presents the Structural Theory of Thermoeconomics a general theory of cost-accounting, that analyzes the different thermoeconomic methodologies and unifies it in a common framework.

The different concepts introduces along the book are illustrated with practical examples, and several thermal plant as a gas turbine cycle for power and steam cogeneration, and a Rankine cycle.

We assume readers have had introductory courses in engineering thermodynamics. For readers with limited backgrounds in thermodynamics s review is provided in Appendix A. Appendix B provides a review of Linear Algebra subjects which are used in the book. Appendix C presents a review of analytical and numerical optimization techniques. The algorithms of the cost-accounting methods presented in the book are explained in detail in Appendix D.

The development and evolution of this books have benefited substantially from the advice of the colleges of CIRCE, the department of Mechanical Engineering of the University of Zaragoza and other researches colleges on thermoeconomics. We also indebted for the comments, questions and revisions of our students of the doctorate course along these years

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The Process of Cost Formation

In this chapter is analyzed in detail the process of cost formation, which has physical roots provided by Second Law of thermodynamics. It is also presented a global outlook of thermoeconomics as a resources saving science connecting physics and economics. The fundamental concepts and tools utilized by thermoeconomics and its scope is fully described

Thermoeconomics is in its widest possible sense the science which connect thermodynamics and economics. An economic analysis can calculate the cost of fuel, investment, operation and maintenance for the whole plant but provide no means to evaluate the single process taking place in subsystem nor how to distribute cost among them. On the other hand, thermodynamic analysis calculates the efficiency of the subsystems and locates and quantifies the irreversibilities but cannot evaluate their significance in terms of the overall production process.

Thermoeconomic analysis combines economic and thermodynamic analysis, by applying the concept of cost. The physical magnitude that connecting physic and economics is entropy generation or more specifically, irreversibility. This represent the “useful” energy or exergy lost or destroyed in a physical process. All real process in a plant are non reversible and as consequence, some exergy is destroyed and some natural resources are consumed and lost forever, which involves a cost in economic terms. All natural resources have an economic cost: the more irreversible a process, the more natural resources consumed and the higher required investment. If one can measure this thermoeconomic cost by identifying, locating and quantifying the causes of inefficiencies in real process, it is possible to provide an objective economic basis using the cost concept.

1.1 Introduction

Nicholas Georgescu-Roegen pointed out in his seminal book, *The Entropy Law and the Economic Process*, that "... the science of thermodynamics began as a physics of economic value and, basically, can still be regarded as such. The Entropy Law itself emerges as the most economic in nature of all natural laws... the economic process and the Entropy Law is only an aspect of a more general fact, namely, that this law is the basis of the economy of life at all levels. ...". Might the justification of thermoeconomics be said in better words?

Since Georgescu-Roegen wrote about the entropic nature of the economic process, no significant effort was made until the 1980s to advance and fertilize thermodynamics with ideas taken from economics. At that time most thermodynamicists were polishing theoretical thermodynamics or studying the thermodynamics of irreversible processes.

But the Second Law tells us more than about thermal engines and heat flows at different temperatures. One feels that the most basic questions about life, death, fate, being and nonbeing, and behavior are in some way related to Second Law. Nothing can be done without the irrevocable expenditure of natural resources, and the amount of natural resources needed to produce something is its thermodynamic cost. All the production processes are irreversible, and what we irreversibly do is destroy natural resources. If we can measure this thermodynamic cost by identifying, locating, and quantifying the causes of inefficiencies of real processes, we are giving an objective basis to economics through the concept of cost.

The search for the cost formation process is where physics connects best with economics, and thermoeconomics can be defined as a general theory of useful energy saving, where conservation is the cornerstone. Concepts such as thermodynamic cost, purpose, causation, resources, systems, efficiency, structure and cost formation process are the bases of thermoeconomics.

Unlike thermodynamics, thermoeconomics is not closed and finished. It is open for new researchers to improve its bases and extend its applications. As in the way thermodynamics was born, thermoeconomics is now closely related to thermal engineering. Cost accounting, diagnosis, improvement, optimization, and design of energy systems are the main uses for thermoeconomics. But thermoeconomics and its content could and should go beyond microeconomic analysis of thermal systems.

Thermoeconomics could one day fulfil the old economists' dream of providing physical roots for economics. It is located in the transition between cost as physical and measurable destruction of resources and cost as analytical accounting of the direct and indirect monetary flows needed to produce a specific product or service.

Thermoeconomics, thus understood, has an integrating and explanatory function. It attempts to integrate and take in the methodologies of energy analysis such as "energy accounting", "embodied energy accounting", "exergy analysis", "emergy analysis", the "analysis of cumulative exergy consumption", "life cycle analysis", "input-output analysis", the "theory of complex energy systems" and "energy optimization", among others. In its analysis it also gives physic-mathematical reasons, or at least attempts to find them, to explain the analogies and discrepancies between the different methodologies. This science gives answers based on the logical application of the second law of thermodynamics in the search for cause-effect relations and chains of causality and finally in a mathematical apparatus common to the conventional economic analysis.

We live in a finite and small world for the people we are and will be, and natural resources are scarce. If we want to survive, we must conserve them. In this endeavour, thermoeconomics plays a key role. We must know the mechanisms by which energy and resources degrade; we must learn to judge which systems work better and systematically improve designs to reduce the consumption of natural resources and we must prevent residues from damaging the environment. These are the reasons for thermoeconomics and its application to engineering energy systems.

1.1.1 Exergy analysis

It is well-known that to have something with a high potential to carry out an action has a great value. In terms of physics this intensive potential is measured using pressure, temperature, high, chemical potential, . . . In economy we use terms as richness, density of information, . . . The real value of the things isn't to have or to know something but to know or to have more than our environment.

If we liberate the bounds that maintain the high potential of our system, it will evolve toward the environment conditions, without producing any useful work. The energy and the mass of our system will not change, but we will note something has change. In the case of the economy the cash flow won't have modified, but my moneybag will.

To quantify these changes the entropy concept is used. The entropy of a system is a function of its intensive potentials, and it is defined in such a way that its entropy increases as much as less useful effects take place when it evolving toward its environment. Therefore, we have introduced the Second Law of Thermodynamics and we have related the concepts of entropy and usefulness.

What is the usefulness from the point of view of the Physics? On one hand, the thermodynamic usefulness or availability always has a reference. The usefulness is measured with regard to a reference level, the more far away the intensive potentials from the reference level are, more usefulness the process has. On the other hand, the magnitude we use in physics to account the availability to do something is the energy (kilowatts per hour). Although, in accordance with the First Principle of Thermodynamic the energy that we pay in our electricity bill is the same one that the heat vanished by a stove that consumes those kilowatts per hour.

Not all the energy types have the same quality, for that reason the thermodynamic usefulness will have to measure in some type of magnitude that indicate quality and quantity at the same time. We could chose the mechanical work or the electric energy as unit.

Therefore we could define the thermodynamic usefulness as the maximum quantity of work that we could obtain from a flow that evolves toward the reference environment, using a ideal machine. We will call it available energy or exergy.

A first law, energy, analysis generally fails to identify energy waste or effective use of fuels and resources. For instance, the first law does not recognize any waste in an adiabatic throttling process, one of the worst processes from the thermodynamic point of view.

The second law of thermodynamics shows that, in some energy carriers (e.g., enthalpy of a flow stream) a part of the energy is useless. Exergy is the part of energy that can be converted into any other form of energy. An exergy analysis based on both the first and second laws of thermodynamics, calculates the useful energy associated with thermodynamic system or with each flow stream in the process.

It also identify and evaluates the inefficiencies of a energy system. This analysis shows that useful energy is destroyed during any step of an energy conversion process, while the total energy remains constant. An energy analysis is the way to unmask the high irreversibilities in process such as combustion, heat transfer throttling or mixing. The causes of irreversibilities or exergy destruction could be located and quantified and the effects of inefficiencies of other components could be detected. Thus, the interdependence of component inefficiencies and the effect of performance deviations from the design conditions can be easily demonstrated. For example any change in the irreversibilities in the mixing device shows in Figure 1 will cause changes in the exergy destruction in the pump and throttling valve, assuming constant conditions for stream 5.

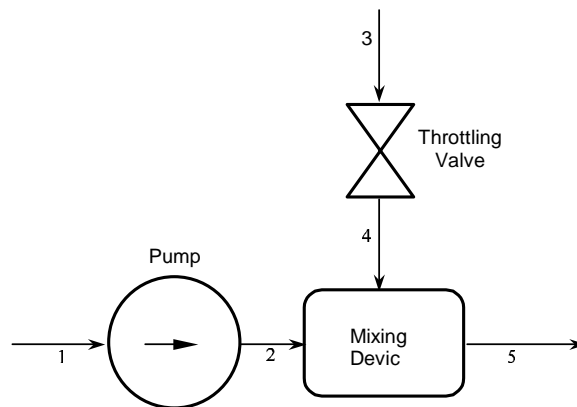


Figure 1.1. Group of components in a energy system

Exergy not only is an objective measure of the thermodynamic value of an energy carrier but also is related closely to the cost of the exergy carrier, because users should pay only for the useful part of energy. Consider two energy carries that consist of the same material (e.g. water) and have the same pressure and the same total energy; one operates at high temperature (superheated steam) an the other at a low temperature (saturated liquid water). The second carrier must have a larger mass. A thermal engineer would probably be willing to pay more for the first carrier than the second one because the cost of the heat exchanger, where the thermal energy would be used, will be lower if the high temperature energy carrier were used instead the lower temperature carrier. This example indicates that exergy rather than mass or energy should serve as a basis for assigning cost to energy carriers.

For the throttling valve shown in Figure 1, the first law of thermodynamics detects no losses in the process from state 3 to state 4. Thus, if we base the cost calculation on the energy content, the cost per unit will be the same before and after the throttling valve, if the capital cost is ignored. The second law of thermodynamics, however quantify the exergy destruction during the throttling process and, with the aid of a cost balance, concludes that the cost per unit of exergy must be higher after the throttling process (flow 4) than before it (flow 3). An increase in the exergy destruction in the throttling valve leads to an increase in the cost per unit of exergy between the valve inlet and the valve outlet and consequently between fuels and final products for the entire system. Thus, the second law sheds light on the cost formation process while the first law could lead erroneous conclusions.

The operation of a pulverized-coal steam power plant offers a third example. If we assume a plant efficiency of 33.33 % and exclude the contribution of the capital cost,

then a unit of electricity will cost three times more than a unit of coal energy. The first law would indicate that the condenser is mainly responsible for this increase while the second law would correctly identify the boiler. The first law cannot reveal that the energy rejected to the condenser has an extremely low exergy content or that the exergy of the superheated steam entering the high pressure turbine is approximately half of the coal exergy. That is, about 50% of the coal useful energy is destroyed in the boiler alone. The values of the first and second law efficiencies of the total plant would be close to equal because the exergy content for both electricity and coal is either identical or close to the corresponding energy content. The first law, however, gives a distorted picture of the losses and cost sources. Similar examples can be found in any energy system.

1.1.2 Irreversibility and exergy cost

How much exergy is dissipated if we break a glass? Almost none is dissipated, because glass is in a meta-stable state near thermodynamic equilibrium with the environment. We cannot save useful energy where none exists. However, if a glass is broken, we make useless all the natural resources used for its production. What is important is not the exergy content of the glass but its exergy cost. Therefore, we will say that the exergy cost of a functional product is the amount of exergy needed to produce it. And a functional product, according to Le Goff "is the product obtained in the energy transformation of its manufacture and defined by the function to which it is destined." The set of manufactured objects that allows the manufacturing of other functional products is named a unit or device. And the procedure for fabricating a functional product from a set of functioning units and from other functional products is named a process or industrial operation. These processes usually produce residues and/or by-products.

Knowing the resources sacrificed in making functional products would be a powerful incentive for optimizing processes. First Law analyses discern as losses only the amounts of energy or materials that cross the boundaries of the system. Friction without energy loss, a spontaneous decrease in temperature, or a mixing process are not considered losses. Second Law ascertains losses in energy quality. Combining both laws allows losses in processes to be quantified and localized. The laws can be combined in many ways. However, production takes materials from the environment and returns products and residues. It is therefore reasonable to analyze exergy, which measures the thermodynamic separation of a product from environmental conditions.

Unfortunately, exergy analysis is necessary but not sufficient to determine the origin of losses. For instance, if the combustion process in a boiler is not well controlled, the volume of air and gases will increase and the fans to disperse them will require additional electricity. The increase in exergy losses from the fans is due to a malfunction of the boiler and not to the fans themselves. Quite commonly, irreversibilities hide costs. Therefore, exergy balances allow localization of losses, but processes and outcomes must also be analyzed. We will term these causality chains processes of cost formation, and their study -an additional step to the conventional exergy analysis - we will term exergy cost accounting.

What is important is not the exergy, E in (kilowatts), that the functional products may contain but the exergy cost E_* , that is the exergy plus all the accumulated irreversibilities needed to get those products.

1.1.3 Thermoeconomic analysis

In a thermoeconomic analysis, we calculate the exergy flow rate associated with each process stream, the irreversibility in each system component, and the exergy (second law) efficiency of each component. In addition to, mass energy and exergy balances, cost balances are formulated for each system component by assigning cost to the exergy of each flow, and whose basic equation is:

$$\dot{C}_P \equiv c_P \dot{E}_P = c_F \dot{E}_F + \dot{Z} \quad (1.1)$$

the cost of the product expressed in monetary units per unit of time, where E_P , E_F are the exergy of the product obtained per unit of time and that of the resources consumed of fuels. Z is the amortization cost of each component in this interval of time and c_F is the unit cost of the exergy of each resource.

With the aid of the cost balances and some auxiliary assumptions, the cost per unit of exergy for each process stream is calculated. Generally this cost is only known for the raw fuel entering the plant from their prices. With a thermoeconomic analysis the cost of all internal flow streams and the final product of the plant are calculated.

The effectiveness in the design or operation of an energy system increases when we understand the real causes and the process of the cost formation. A thermoeconomic analysis identifies these causes. This information, complemented by the engineer's experience, helps to reduce the product costs in energy systems. Decisions about design, operation and repair and replacement of equipment are facilitated. In addition thermoeconomic analysis provides an objective cost allocation to more than one product of the same process, as for instance a cogeneration which produces electricity and steam for heat. Finally, thermoeconomics helps management to decide how to allocate research and development funds to improve plant components that contribute most significantly to the product cost.

At present, thermoeconomic analysis could be applied to solve different problems in complex energy systems, as for instance:

- Rational price assessment of plant product based on physical criteria.
- Optimization of specific process unit variables to minimize final product cost, i.e. local and global optimization:
- Detection of inefficiencies and calculation of their economic effects in operating plants, i.e. plant operation thermoeconomic diagnosis
- Evaluation of various design alternatives or operation decision and profitability maximization
- Energy audits.

Moreover, thermoeconomics and its applications should go beyond microeconomic analysis for optimization of energy systems. It should be providing the physical roots for economics, giving thermodynamic and mathematical answers to all methodologies of energy analysis used up to present day, and be the science which underlies all of them.

The Exergy Cost Theory

2.1 The Thermoeconomic Model

2.1.1 Definitions and concepts

To illustrate the different concepts, we will use a simple example of a thermal system, which is a cogeneration plant based on a gas turbine.

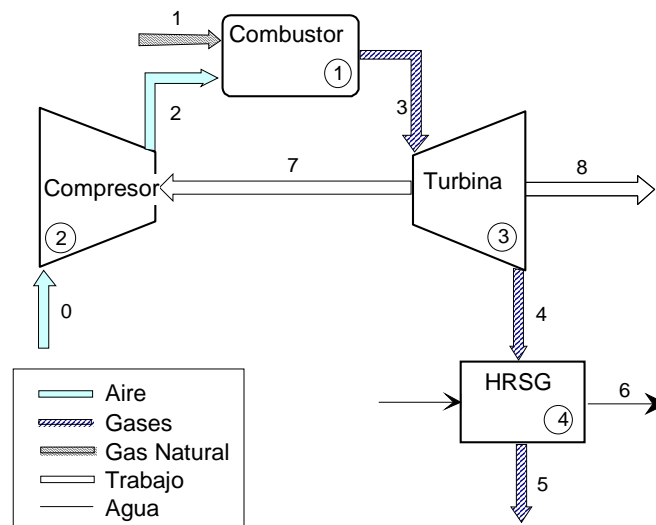


Figure 2.1. Plant layout of the cogeneration plant.

Industrial installations have a defined aim, to produce one or several products. The quantity of resources is identified through mass or energy flows, which are known as fuel. Each of the components of the plant also has a well-defined objective characterized by its fuel and its product, as figure 1 shows for the cogeneration plant.

To carry out a thermoeconomic analysis of a system, it is necessary to identify its flows with a magnitude sensitive to the changes in quality and quantity of the energy processed. Exergy is an adequate magnitude because it expresses

the thermodynamic separation of the intensive properties characterizing the flow (p_j, T_j, μ_j) with respect to those of the environment $(p_0, T_0, \mu_{j,00})$. The thermo-economic methodology explained in this book is based on this fact. However, other magnitudes are possible. In fact, this problem is currently under study.

Accordingly, it can be said that fuel F is the exergy provided for the process through the resources and product P is the exergy that contains the benefits obtained. Thus, for the cogeneration plant in our example, the exergy of process steam and the net power are the products, and the exergy provided by the natural gas is the fuel. In table 1 is shown the Fuel and product for all the component or process units of the plant.

The aim of the combustion is to increase the exergy of the air flow E_2 that exits from the compressor. The product is, therefore, the difference of exergy between flows 3 and 2, and the exergy of natural gas is consumed as fuel. In the case of the turbine, the aim is to obtain mechanical energy, therefore, the product is the exergy employed to drive the compressor E_5 and the net power of the plant E_6 . The exergy provided by the gas expanded in the turbine $E_3 - E_4$ is the fuel.

Table 2.1. Fuel & Product definition for the analyzed system

#	Process unit	Fuel	Product
1	Combustor	E_1	$E_3 - E_2$
2	Compressor	E_5	$E_2 - E_0$
3	Turbine	$E_3 - E_4$	$E_5 + E_6$
4	HRSG	E_4	E_7
Total plant		E_1	$E_6 + E_7$

Note that the product of the combustion chamber, like the fuel of the turbine is formed by the flows entering and leaving the processes. That is to say, the fuel does not consist exclusively of flows entering the system, nor do the products consist exclusively of flows leaving the system. In the fuel and product definition we will distinguish between continuous exergy streams and interrupted or input/output streams.

We also define losses L as those flows that leave the unit and the plant, are not subsequently used, and do not require a special treatment. When these flows leave the unit, exergy dissipates into the environment. If we suitably enlarge the limits of the unit, these external irreversibilities become internal. We will call the irreversibility (I) of the unit the sum of internal exergy destructions plus losses occurring in it, $I = L + D$.

We will call productive units those whose objective is to transfer the exergy contained in the fuels to the products. The fuel-product definitions for productive units should be chosen such that the equation $F - P = I$ is an expression of each exergy balance. The exergy efficiency of these units is defined as:

$$\varepsilon = \frac{\text{Exergy of useful product}}{\text{Exergy of resources}} = \frac{P}{F} \quad 0 < \varepsilon < 1$$

The inverse is the unit exergy consumption defined as:

$$\kappa = \frac{F}{P} \quad \kappa > 1$$

from a formal point of view a system can be considered as a complex entity made up of a collection of components and of the relationship existing between them and their environment. Thus, an energy system, such as the analyzed cogeneration plant of figure 1 can be represented as a collection of components interrelated through the mass and energy flows, which behavior is analyzed using a physical model with a set of equations to describe the physical behavior of the process units. It calculates parameters such as temperatures, pressures, efficiencies, power generated, etc. to describe the physical state of the plant.

Depending on the depth of the analysis, a decision has to be taken on the required detail i.e., which flows and process units are to be considered. Various parts of the installation can be combined into one process unit and physical units can be further disaggregated. The disaggregation level is interpreted as the subsystems that compose the total system. Each subsystem can be a part of a piece of equipment, the piece of equipment itself, or a group of pieces of equipment. The same can be said for the interacting energy flows. The disaggregation level provides a breakdown of the total irreversibility among the plant components. The chosen disaggregation level will affect the conclusions of the analyses. In fact, if we do not have more information about the system than that defined by its disaggregation level, we cannot demand from the obtained set of costs more information than we have introduced. Conversely, the analyst, not the theory, should be required to disaggregate the plant, looking for cause until the information can be used effectively.

It is important to choose an appropriate aggregation level that properly defines the behavior of each process unit and its purpose in the overall production process. The physical structure (see figure 1.1) depicts the process units, mass stream and connecting energy flows considered in the physical model.

Usually the information embedded in the physical model comes from a set of real data provided by the Data Acquisition System of a plant, or by a plant simulator. Those measured physical data are temperatures, pressures, mass flow rates and compositions of all mass flows together with the heat and power rates of the energy flows considered. Therefore, the finally chosen aggregation level should take into account that the thermoeconomic analysis will start from those real measured data in a real plant.

Nevertheless, when performing a thermoeconomic analysis, it is absolutely necessary to define a thermoeconomic model of the plant, which considers the productive purpose of the process units, i.e. the definitions of fuels and products and the distribution of the resources throughout the plant. The productive model can be graphically depicted obtaining the *Fuel/Product Diagram* also called *Productive Structure* or *Functional Diagram*. In this scheme, the flows (lines connecting the equipment) are the fuel and the product of each subsystem. Each piece of equipment in the plant has an outlet flow (product) and, at least, an inlet flow (fuel).

The capital cost of the units is also considered as an external plant resource and is represented as inlet flows coming directly from the environment (not considered in figure 2). Since the fuel of a process unit can be the product of another and the product of a process unit can be the fuel of several subsystems. The productive structure is a graphical representation of resource distribution throughout the plant. Thus, the devices providing exergy to the working fluid air/gases in the cogeneration plant of figure 1.1 are the compressor and the combustor. The turbine and the HRSG consume the exergy provided by compressor and combustor. This is represented in the productive structure by the junction, in which the product of compressor and combustor are joined, and is distributed to the turbine and HRSG..

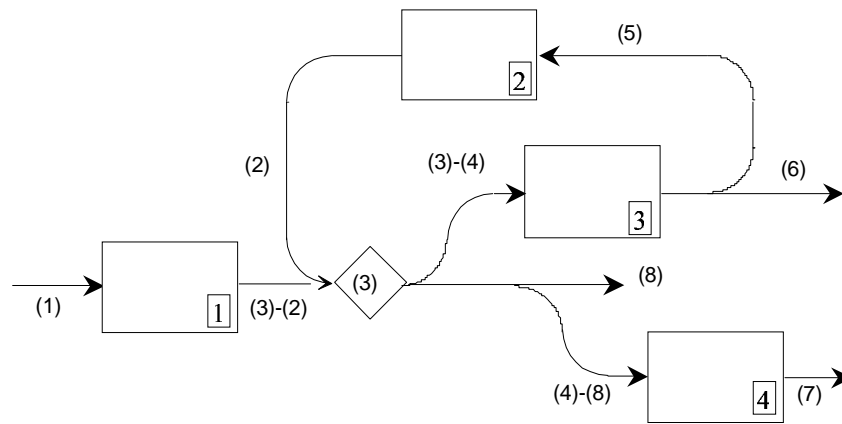


Figure 2.2. Productive structure of the cogeneration plant.

Note that in most of cases the flows appearing in the productive structure are fictitious and are not necessarily physical flows. While each plant has only one physical structure to describe the physical relations between the process units, various productive structures can be defined depending on the fuel and product definitions as well as decisions on how the plant resources are distributed among the process units. Figure 2 shows the productive structure corresponding to a specific thermoeconomic model, i.e. corresponding to a specific Fuel-Product definition (see table 1.1). Depending on the thermoeconomic model definition the productive structure varies, obtaining thus as many productive structures as thermoeconomic model definitions.

Figure 1 shows a convenient disaggregation level of the analyzed cogeneration plant just for presenting ideas. The flows have been numbered as follows: (i) The flow of air into the compressor has been eliminated because its energy and exergy are zero. (ii) We consider a flow of process steam (flow 7) with an exergy value (E_7) equal to the difference of exergy between the flow of steam produced and the flow of feed water entering the HRSG. (iii) In the HRSG, the flow corresponding to the outlet gases has been removed because its exergy is not used later and the stream is exhausted into the atmosphere. The same reason applies to combustor heat losses.

2.1.2 Cost accounting

According to the management theory, cost accounting is an economic task for recording, measuring, and reporting how much things cost. Companies and individuals tend to optimize costs because cost is a loss of resources, and problems generally appear when appropriate insight of costs and their causes are lacking. Business managers use cost data for decision making and performance evaluation and control. They have techniques for costing products and services and use differential costs for estimating how costs will differ among the alternatives. Managerial cost accounting became a profession many years ago, and almost every organization uses it.

Energy cost accounting is in addition to a managerial technique for keeping low the use of energy resources, provides a rationale for assessing the cost of products in terms of natural resources and their impact on the environment and helps to optimize and synthesize complex energy systems. Since 1985 the author of this article and coworkers have been developing the exergy cost theory (ECT) and its

applications to answer these problems. A simplified description of the theory is now presented.

In any energy system the exergy of the resources is greater than or equal to that of the products. For the plant as a whole as well as for any unit, $F - P = I > 0$. The amount of exergy needed to obtain the products is equal to the exergy of the resources consumed. This idea permits the introduction of a new thermodynamic concept called exergy cost, that is defined as follows: *given a system whose limits, disaggregation level, and production aim of the subsystems have been defined, we call exergy cost, E^* , of a physical flow the amount of exergy needed to produce this flow.*

The exergy cost is a thermodynamic function, like exergy (E), and its definition is closely related to others that are common in literature, such as materials' energy content, embodied energy, and cumulative exergy consumption.

The cost of a flow is an emergent property, that is, it does not exist as a separate thermodynamic property of the flow. Cost is always linked to the production process. And this process links a set of internal and external flows. Therefore, not only the cost of a flow but also a complete set of interrelated costs need to be determined. On the other hand, the classification of flows as internal or external depends on the system limits as for the case of subsystems or the system itself.

When performing a cost analysis in a system we can distinguish between *average costs*, which are ratios and express the average amount of resources per unit of product, and *marginal costs*, which are a derivation and indicate the additional resources required to generate one more unit of the product under specified conditions. Mathematically they are defined as:

$$\text{Unit average cost: } \bar{k}_i^* = \frac{E_i^*}{E_i} \quad (2.1a)$$

$$\text{Unit marginal cost: } \lambda_i = \left(\frac{\partial E_0}{\partial E_i} \right)_{\text{cond}} \quad (2.1b)$$

The average costs are only known after production, when we know how many resources were used and the production has already been obtained. The average cost is not predictive. Knowing the average unit cost of a product does not provide the cost of a production process $E_i + \Delta E_i$. Thermoeconomic cost accounting methods calculate average exergy costs and use them as a basis for a rational price assessment, under physical criteria, of the internal flows and the products of the plant.

Marginal costs are predictive in nature. They can be used to calculate additional fuel consumption when the production is modified. Thermoeconomic optimization methods are based on marginal costs when solving optimization problems.

2.1.3 Calculation of average exergy cost

The fundamental problem of cost allocation can be formulated as follows: *Given a system whose limits have been defined and a disaggregation level that specifies the subsystems that constitute it, how do we obtain the costs (average costs) of all the flows that become interrelated in this structure?*

An initial procedure to solve it, can be based on the next four propositions.

P1 Rule: The exergy cost is relative to the resource flows. In the absence of external assessment, the exergy cost of the flows entering the plant equals their exergy. In

other words, the unit exergy cost of resources is one. As many equations can be formulated $E_i^* = E_i$ as flows entering the plant.

P2 Rule: The exergy cost is a conservative property. For each component of a system the sum of the exergy costs of the inlet flows is equal to the sum of the exergy costs of the exiting flows. In matrix form, and in the absence of external assessment, the exergy cost balance for all compounds of the plant is $\mathbf{A}\mathbf{E}^* = 0$, where \mathbf{A} is the incidence matrix ($n \times m$), which relates for whatever system, no matter how complex, the components n with the flows m of the system, and \mathbf{E}^* is a vector ($m \times 1$) that contains the exergy cost of the flows. This equation provides as many equations for calculating the exergy costs as the number of components in the installation.

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

We now reconsider the problem of cost allocation. If one wants to calculate the cost of each of the m flows relevant to the disaggregation level considered for analysis, it will be necessary to write m independent equations. If all units have only one output flow that is not classified as a loss flow, then the problem is solved by applying the stated propositions. In this case, we say that the system or process analyzed is sequential. In the opposite case, additional equations must be written for each unit equal to the number of output flows that are not loss flows minus one. At this point we need to use exergy to reasonably allocate costs, because this property enables us to compare the equivalence of the flows according to the principles of thermodynamics. Note also that in any structure the number of bifurcations x equals the number of flows minus the number of units and the number of resource flows ($x = m - n - e$), allowing us to associate the problem of cost allocation to bifurcations. The additional propositions are as follows:

P3 Rule If an output flow of a unit is a part of the fuel of this unit (non-exhausted fuel), the unit exergy cost is the same as that of the input flow from which the output flow comes.

P4 Rule If a unit has a product composed of several flows with the same thermodynamic quality, then the same unit exergy cost will be assigned to all of them. Even if two or more products can be identified in the same unit, their formation process is the same, and therefore we assign them a cost proportional to the exergy they have.

The validity of these propositions is proved in an next chapter, which also provides the way for obtaining new ones in complex cases. However, in many cases the application of these propositions is a matter of disaggregation until we can recognize units with products of the same equality and exit flows identified as non exhausted fuels.

We now consider the case of the cogeneration system in figure 1. The plant shows $n = 4$ units and $m = 7$ flows. We assume that the exergy of these flows is known. Propositions P1–P4 offer a rational procedure for determining the exergy costs of the m flows of the system. The proposition P1 says that the exergy costs of the e flows that enter the plant coincide with their exergy, thus providing e equations. The fuel flow to the combustion chamber gives the equation $E_1^* = E_1 = \omega_1$, (or $k_1^* = 1$) (ω_1 , denotes a given datum). Note that the air entering the compressor should provide another equation. However, the exergy of air is zero, and the air is taken from the environment at no cost. The proposition P2 says that a balance of exergy costs can be established for each unit; therefore, n equations are available, as many as components. These will be four for the cogeneration system of our example.

$$\begin{bmatrix} \mathbf{A} \rightarrow \\ \dots \\ \alpha_k \rightarrow \\ \dots \\ \alpha_x \rightarrow \\ \dots \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & -X_{3F} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X_{3P} & 1 & 0 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ \dots \\ E_5^* \\ \dots \\ E_6^* \\ E_7^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dots \\ \square \omega_1 \\ \dots \\ 0 \\ 0 \end{bmatrix}$$

Figure 2.3. Incidence matrix of the cogeneration system.

The propositions P3 and P4 yield as many cost equations as the number of bifurcations in the plant $x = m - n - e$. Two bifurcations are in the cogeneration system, appearing both in the turbine, one corresponding to the exiting stream whose exergy has not taken part in the process and corresponds to a non-exhausted fuel. Consequently, proposition P3 must be applied to this case, resulting in the following equation:

$$E_4/E_4 = E_3^*/E_3$$

Alternatively, if we define $x_{3F} = -E_4/E_3$ as the bifurcation exergy ratio corresponding to the fuel $E_3 - E_4$ of the turbine, then we get the equation:

$$-x_{3F}E_3^* + E_4^* = 0$$

The other bifurcation of the turbine corresponds to the output flows 5 and 6 and constitutes its product; therefore, we will apply proposition P4, which results in the following equation:

$$E_6^*/E_6 = E_5^*/E_5$$

If we define $x_{3P} = E_6/E_5$ as the bifurcation exergy ratio corresponding to the product $E_5 + E_6$ of the turbine; then:

$$-x_{3P}E_5^* + E_6^* = 0$$

Now we have the m equations required for the calculation of the exergy flows of the cogeneration system. In matrix form can be written as shown in figure 4. In general form this system can be expressed as:

$$[\mathbf{A}] \cdot \mathbf{E}^* = \mathbf{Y}_e^* \quad (2.2)$$

where $[\mathbf{A}]$ is the cost matrix ($m \times n$) composed for n rows corresponding to the incidence matrix \mathbf{A} ($n \times m$) of the plant, and $m - n$ rows corresponding to the production matrix α ($m - n \times m$). $\mathbf{Y}_e^*(m \times 1)$ is the vector of external assessment composed of n elements as a result of proposition P1, e elements with the actual values of the exergies (ω_e) corresponding to the resources of the plant (*P2 rule*), and $m - n - e$ null elements corresponding to the fuel and product bifurcations (*P3 - P4 rule*).

Inverting matrix $[\mathbf{A}]$ allows for the exergy costs to be obtained through the equation:

$$\mathbf{E}^* = [\mathbf{A}]^{-1} \mathbf{Y}_e^* \quad (2.3)$$

2.1.4 Calculation of exergoeconomic costs

Calculating the monetary cost of the internal flows and final products in thermal or chemical plants is a problem of the utmost importance, as the monetary cost is directly linked to the production costs of the different components of the productive process. For these plants the formation of the economic cost of the internal flows and final products is related to both the thermodynamic efficiency of the process and to the depreciation and maintenance cost of the units. Therefore, one can define the exergoeconomic cost of a flow as the combination of two contributions: the first comes from the monetary cost of the exergy entering the plant needed to produce this flow, that is its exergy cost, and the second covers the rest of the cost generated in the productive process associated with the achievement of the flow (capital, maintenance, ...).

Accordingly, we call exergoeconomic cost of a flow the quantity of resources, assessed in monetary units, needed to obtain this flow, and we denote it by \mathbf{C} . In the same way we call exergoeconomic cost of the fuel (product) the economic resources necessary to obtain the fuel or product of the component, and we denote it by \mathbf{C}_F or \mathbf{C}_P . If Z_i is the levelized cost of acquisition and maintenance, of the component i in €/s, the exergoeconomic cost balance for this component can be written as:

$$\mathbf{C}_F + \mathbf{Z} = \mathbf{C}_P \quad (2.4)$$

where vector \mathbf{Z} ($n \times 1$) contains the levelized acquisition cost of the plant components and \mathbf{C}_F and \mathbf{C}_P are ($n \times 1$) vectors containing the exergoeconomic costs of the fuel and products of the components. Considering the previous equation for all units of the plant we obtain the set of equations: $\mathbf{A}\mathbf{C} = -\mathbf{Z}$ which corresponding to the cost balance rule P2, and the unknown quantities C_i are the exergoeconomic costs of the n flows. As for the exergy costs $m - n$ auxiliary equations are required to find the exergoeconomic costs of the flows. The auxiliary equations can be formulated using the P1, P3 and P4 rules. We conclude that the mathematical problem of calculating exergoeconomic costs of the flows of a plant requires solving the system of m equations with m unknowns:

$$[\mathbf{A}] \cdot \mathbf{C} = \mathbf{Z}_e \quad (2.5)$$

where $\mathbf{Z}_e = {}^t[-\mathbf{Z}|\mathbf{C}_e|0]$ is the vector that contains the external economic assessments.

The uniqueness of the matrix of costs, when applied to the calculation of the exergy and exergoeconomic costs, reflects the fact that passing from the former to the latter simply involves modifying the units in which the production factors are expressed (kilojoules or euros).

2.1.5 External assessment and additional concepts

Up to now, the system or plant has been considered since a thermodynamic point of view, without allowing for the physical or economic relationships with other systems or plants. The effects of these relations on costs can be introduced into the analysis by modifying the external assessment vector. In any case, the matrix of costs will remain unaltered. Some important cases are the following:

Exergy Amortization: In the balance of exergy costs, it has not been considered the fact that the units that form an installation are functional products and therefore have their own exergy cost. To keep them in good operation, additional exergy will be required. After determining the exergy costs of the units, it will be necessary to distribute these costs over the total working lives. In this way, it is possible to obtain with conventional methods a vector of dimension n that corresponds to

the exergy amortization of the units. In a parallel manner, the vector of exergy maintenance will be obtained. By defining the vector sum of both as $\mathbf{Y}^* = \mathbf{Y}_A^* + \mathbf{Y}_A^*$, it is possible to reformulate the balances of the exergy cost of the installation (P2 proposition) in general as $\mathbf{A}\mathbf{E}^* = -\mathbf{Y}^*$.

Residues: Residual flows require an additional expense of resources for disposal. An example is the flows of slag and fly ash in a coal boiler, which require power-operated units without which the plant could not work. Thus, removing these flows from a coal boiler entails an exergy cost that equals R^* . Parallel to the process of product formation, there exists a process of residue formation. Residues obviously are not formed in the last device they pass through. For instance, the cost of stack gases in a boiler should not be allocated to the stack but to the combustion chamber. For a proper allocation of the costs of residues we must follow their formation process and assign the cost of their disposal to the unit in which they are formed. The residual structure has its own units-and-flows representation, but the connecting arrows go in the direction opposite to the physical flows to make explicit their negative cost character.

Assessment of the plant fuels: The fuel flows consumed by an industrial installation are rarely composed of non transformed primary resources (fuels, metals, geothermal deposits, etc.) whose values are represented by their thermodynamic disequilibrium with the reference environment, that is to say, from their exergy. Thus, the coal processed by a boiler has an exergy cost of primary resources V^* , which are higher than the coal's exergy because of different processes: extraction, storage, transport, etc. If we want to incorporate their contribution to the exergy costs of the flows and products of the plant into our analysis, we must apply the proposition P2 to coal flow in the following form: $\omega = V^*$.

Cumulative exergy cost or ecological cost: The vector \mathbf{Y}^* incorporates the external information that finally determines the exergy costs of an installation. In a conventional assessment, we distinguish between the thermodynamic system that constitutes the installation and its thermodynamic environment, ignoring completely the irreversibilities that take place there and that form part of the process using the primary resources to generate the final products. As these assessments are being incorporated, the exergy costs will include a greater part of the external irreversibilities. The latter have their origin in the manufacture, installation, repair, and maintenance of the units \mathbf{Y}^* , in the elimination of residues R^* , and in the previous production of the flows entering the plant V^* . Logically, the most appropriate external assessment will depend on the aim of the analysis that is carried out. The natural assessment of vector \mathbf{Y}^* consists of considering each and every one of the external irreversibilities, and, therefore, \mathbf{Y}^* , R^* , and V^* reflect their costs of primary resources.

This type of analysis is of foremost importance when considering the ecological cost of products used by our society, and a systematic accounting of each and every ecological cost of products would lead an answer to the viability of our technology and the sustainability of our society.

2.2 Closure

There is nothing to stop the flows, which characterize the interactions between the subsystems, representing only energy. They may also be monetary interactions, e.g. the costs of amortization of subsystems; or information interactions, e.g. control signals; or time interactions, e.g. the time it has taken to produce a certain product. The Leontieff's technical coefficients themselves have been used to view

the impact on CO_2 , SO_2 , solar energy, etc.. In other words, and as it is well known, the input-output theory can be applied in many ways and generalized.

However, what characterizes exergy costs beyond the other costs, which can be defined within a system?

In the opinion of the author it is its natural relationship with the second law which is as we have seen that which takes into account the real losses that are to be accounted in a system -internal or external or losses of quality-.

In order to connect exergy with exergy costs there has to be, as we have already said, a conceptual leap of the highest importance. And it is that of classifying the flows in fuels or resources, F , products, P and residues, R , together with breaking down the system enough to be able to recognize the linearity in each of the subsystems that make it up. Then and only then does the second law becomes into: $F - P = I$. This equation is something else than the second law, i.e. than the exergy balance.

If the efficiency, or its inverse, are constants in a range of work, then its own definition serves as a characteristic equation. That is to say $F = k P$, which combining with previous equation we get $I = (k - 1)P$, which relates irreversibility to the unit exergy consumption and production. Or, in the words of the input-output theory, *it relates the technical coefficients to the true destruction of available energy, which takes place in any production process*. That is to say, it relates thermodynamics to economics but without leaving the field of physics as it still does not employ monetary units. A physical primal imposes a physical dual of costs. Those costs are of physical nature not related with the vagaries of economics nor prices or values.

The exergy cost relates local irreversibility to the global consumption of physical resources entering into a system. With a system well broken down into, and with a production structure, which represents its global behavior in a sufficiently accurate way it would be possible to optimize a system locally without the need to take into account interactions with other subsystems.

The exergy cost relates local irreversibility to the global consumption of physical resources entering into a system. With a system well broken down into, and with a production structure, which represents its global behavior in a sufficiently accurate way it would be possible to optimize a system locally without the need to take into account interactions with other subsystems. The exergy destruction will be more costly when the more advanced this will be in the production chain. *Cost, thermodynamic efficiency and irreversibility remain linked together in a single theoretical body.*

Symbolic Exergoeconomics

Symbolic thermoeconomics is a methodology for the analysis of the productive structure, and the natural resources consumption process in energy systems. It appears as a technique, based on the Exergy Cost Theory, to obtain general equations, which relate the overall efficiency of an energy system and other thermoeconomic variables as fuel, product, exergy cost, with the efficiency of each component which forms it. By mean of the equations obtained, it is possible to analyze the influence of the individual consumption of each component on the total amount of external resources required to obtain a product. Therefore, the productive structure and the cost formation process of the products are explained.

The cost accounting methodologies, as Exergy Cost Theory, propose methods to determine the amount of resources required for obtaining a product. They are based on cost assessment rules, which attribute to the useful product the resource cost of each component, and distribute its costs proportionally to its exergies. They are mainly numerical techniques that calculate the cost values in an accurate way, by solving sets of linear equations, but they cannot identify the causes of the cost formation process.

Suppose the thermal system shown in Figure 1. In this article it is explained the procedure for obtaining the global efficiency of the system as a function of the efficiencies, ζ_i , of its components is:

$$\epsilon_T = \frac{\epsilon_1 (\epsilon_2 y_2 (1 - y_3) + (1 - y_3) \epsilon_3)}{1 - y_2 y_3 \epsilon_2 \epsilon_3}$$

where terms y_i represents the bifurcation ratios.

This poses some initial questions: Could we obtain such a formula in a general way? What conditions must fulfill our method to assess it? If such a formula exists, questions of the type, *what happens if?*, could be simplified. Thus we could compute, for example, how a variation in the efficiency of a component modify the efficiency of the whole plant. With general formulae we can get general solutions to general problems. The proposed analysis provides a set of valuable tools for the cost accounting, diagnosis, optimization and synthesis of energy systems.

Symbolic computation packages, like *Mathematica* or *MatLab*, could be used to solve a wide variety of technical computing problems, such as to obtain both

analytical and numerical solutions of linear systems of equations. If we bring together Exergy Cost Theory and Symbolic Computation then it will be possible to find out a general way to obtain formulae like that shown above. This was the reason to call this methodology *Symbolic Thermoeconomics*.

3.1 The Fuel-Product Model

The first stage to identify the cost process formation consists of building, from the physical structure of the plant, a productive scheme which shows where the product of each component is used and the origin of the resources of each component.

The problem of the productive structure identification is closely related to *Leontief's input-output economic analysis*. It consists of a qualitative and quantitative analysis of the relations that link the flows of goods and services between the components of an economic unit, in order to study its structural characteristics.

An equivalent model could be applied to thermal systems. It can be represented by a fuel/product diagram, such as shown in Figure 1.2 and Table 1, for the plant of the example. It is also called in other thermoeconomic methodologies Functional Diagram. Note that, not all products of components #1 and #2 are used as resources in components #3, #4 or final product, the flow #8 is a residue generated in components #1 and #2.

Table 3.1. Fuel Product Table

	F_0	F_1	F_2	F_3	F_4	Total
P_0	0	E_1	0	0	0	E_1
P_1	0	0	0	$r_1(E_3 - E_4)$	$r_1(E_4 - E_8)$	$r_1(E_3 - E_8)$
P_2	0	0	0	$r_2(E_3 - E_4)$	$r_2(E_4 - E_8)$	$r_2(E_3 - E_8)$
P_3	E_6	0	E_5	0	0	$E_5 + E_6$
P_4	E_7	0	0	0	0	E_7
Total	$E_6 + E_7$	E_1	E_5	$E_3 - E_4$	$E_4 - E_8$	

In table 1 we have used some exergy junction ratios defined as:

$$r_1 = \frac{E_3 - E_2}{E_3} \quad r_2 = \frac{E_2}{E_3} \quad r_1 + r_2 = 1$$

In accordance with this model the production of one component is used as fuel of another component or as a part of the total production of the plant:

$$P_i = E_{i0} + \sum_{j=1}^n E_{ij} \quad i = 0, 1, \dots, n \quad (3.1)$$

where E_{ij} is the production portion of the i -th component that fuels the j -th component. In the above expression, we consider the component 0, as the system environment, then E_{i0} represents the production portion of the component i which leads to the final product, coming from the environment to the component i .

On the other hand, the resources entering each component, could be expressed as:

$$F_i = E_{0i} + \sum_{j=1}^n E_{ji} \quad i = 0, 1, \dots, n \quad (3.2)$$

where E_{0i} represents the external resources entering to the plant, which go into the i -th component. Therefore, the total fuel and product of the system could be

expressed as:

$$\begin{aligned} F_T &\equiv P_0 = \sum_{j=1}^n E_{0j} \\ P_T &\equiv F_0 = \sum_{j=1}^n E_{j0} \end{aligned} \quad (3.3)$$

3.1.1 Cost Model Equations

The proposed model could be also applied to exergy cost:

$$P_i^* = E_{i0}^* + \sum_{j=1}^n E_{ij}^* \quad i = 0, 1, \dots, n \quad (3.4)$$

$$F_i^* = E_{0i}^* + \sum_{j=1}^n E_{ji}^* \quad i = 0, 1, \dots, n \quad (3.5)$$

According with the proposed model, the cost assessment rules of the Exergy Cost Theory, explained in the previous chapter can be written as:

P1: The cost of the external resources is known and equal to its exergy:

$$E_{0i}^* = E_{0i} \quad (3.6)$$

P2: The exergy cost of Fuel is equal to the exergy cost of Product:

$$P_i^* = F_i^* \quad (3.7)$$

P3: The exergy cost of the flows produced in a component is proportional to their exergy, therefore:

$$k_{ij}^* = k_{p,i}^* \quad (3.8)$$

where k_{ij}^* represents the unitary costs of the flow E_{ij} , and is defined by the relation:

$$E_{ij}^* = k_{ij}^* E_{ij} \quad (3.9)$$

The previous relations could be combined used by means of the equation:

$$P_i^* = E_{0i} + \sum_{j=1}^n k_{p,i}^* E_{ji} \quad (3.10)$$

It let us to determine the exergy cost of the products P_i^* , by solving the set of linear equations given by:

$$P_i^* - \sum_{j=1}^n \frac{E_{ji}}{P_j} P_j^* = E_{0i} \quad (3.11)$$

or in an equivalent way, for the unit exergy cost:

$$k_{p,i}^* - \sum_{j=1}^n \frac{E_{ji}}{P_i} k_{p,j}^* = k_{0i}^* \quad (3.12)$$

Example

The cost of the product in the gas turbine plant, is giving as the solution on the following set of linear equation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -y_2 & 0 \\ -y_3 & -y_3 & 1 & 0 \\ -y_4 & -y_4 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where the coefficients of the matrix are:

$$y_2 = \frac{E_5}{E_5 + E_6} = 0.5435$$

$$y_3 = \frac{E_3 - E_4}{E_3 - E_8} = 0.6268$$

$$y_4 = \frac{E_4 - E_8}{E_3 - E_8} = 0.3732$$

The numerical solution of this system of equations is shown in Table 2.3.

3.2 The \mathcal{FP} Representation

In the next sections, we are going to develop a general method to obtain analytical formulas of the efficiency and production costs of a thermal system, using the relationships of the fuel-product model. Therefore, we need to identify which variables allow relating the total production of the system with the external resources required, or the total cost of the resources required for obtaining a production objective.

3.2.1 Distribution Ratios

We define *distribution* coefficient y_{ij} , as the portion of the production of the j -th component used as resource in the i -th component:

$$y_{ij} = \frac{E_{ji}}{P_j} \quad (3.13)$$

and verifies, the sum of all distribution ratios of a component must be equal one:

$$\sum_{i=0}^n y_{ij} = 1$$

Therefore Eqn (2.2) could be written as:

$$F_i = E_{0i} + \sum_{j=1}^n y_{ij} P_j \quad i = 1, \dots, n \quad (3.14a)$$

$$P_T = \sum_{j=1}^n y_{0j} P_j \quad (3.14b)$$

It shows the fuel of each component as a linear function of the products that form it. The previous equation could be written in matrix form as:

$$\mathbf{F} = \mathbf{F}_e + \langle \mathbf{FP} \rangle \mathbf{P} \quad (3.15)$$

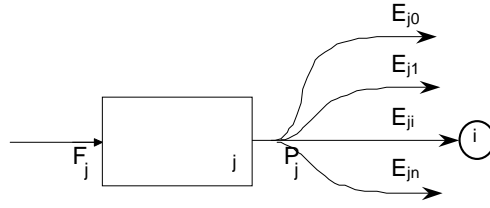


Figure 3.1. Definition of distribution ratios

The vector $\mathbf{F}_e \equiv (E_{01}, \dots, E_{0n})$ contains the exergy values of the external resources for each component, and the $\langle \mathbf{FP} \rangle$ is a $(n \times n)$ matrix whose elements are the distribution coefficient y_{ij} .

In the case of the exergy cost, we can obtain an equivalent relationship:

$$F_i^* = E_{0i} + \sum_{j=1}^n y_{ij} P_j^* \quad i = 1, \dots, n \quad (3.16a)$$

or in matrix form as:

$$\mathbf{F}^* = \mathbf{F}_e + \langle \mathbf{FP} \rangle \mathbf{P}^* \quad (3.16b)$$

Note that the distribution coefficient matrix is the same for exergy and exergy cost, due to the assessment cost rules, which make the production cost proportional to its exergy.

3.2.2 Cost and Efficiency Formulae

Now, we are going to obtain general formulas to relate the efficiency and product costs of the plant with the efficiency of each component.

The efficiency or unit exergy consumption of each component k_i is defined as the amount of resources, measured on exergy, required to obtain a unit of product. This could be written in matrix form as:

$$\mathbf{F} = \mathbf{K}_D \mathbf{P} \quad (3.17)$$

where \mathbf{K}_D is a diagonal matrix $(n \times n)$ which contains the unit exergy consumption of each component k_i . Therefore, from Eqns 10, 12, we get:

$$(\mathbf{K}_D - \langle \mathbf{FP} \rangle) \mathbf{P} = \mathbf{F}_e \quad (3.18)$$

the $\mathbf{K}_D - \langle \mathbf{FP} \rangle$ is a diagonal strictly dominant matrix. Hence, it has inverse matrix whose elements are all positive values.

Therefore, the previous equation represents a system of linear equations that let us express the production of each component as a function of:

- The external resources: \mathbf{F}_e
- The efficiency of each component: \mathbf{K}_D
- The distribution parameters: $\langle \mathbf{FP} \rangle$.

In a general way, given a generic exergy variable \mathbf{X} , as fuel exergy, product exergy, exergy cost of product, there is a linear operator, a matrix ($n \times n$), whose elements are a function of the efficiencies of the components and the distribution parameters, that transform the external resources into each variable:

$$\mathbf{P} = \langle \mathbf{P} | \mathbf{F}_e \quad \text{where} \quad \langle \mathbf{P} | = (\mathbf{K}_D - \langle \mathbf{FP} \rangle)^{-1} \quad (3.19a)$$

$$\mathbf{F} = \langle \mathbf{F} | \mathbf{F}_e \quad \text{where} \quad \langle \mathbf{F} | = \mathbf{K}_D \langle \mathbf{P} | \quad (3.19b)$$

$$\mathbf{I} = \langle \mathbf{I} | \mathbf{F}_e \quad \text{where} \quad \langle \mathbf{I} | = (\mathbf{K}_D - \mathbf{U}_D) \langle \mathbf{P} | \quad (3.19c)$$

Applying the cost balance Eqn 3, to expression (2.16), the following holds:

$$\mathbf{P}^* = \langle \mathbf{P}^* | \mathbf{F}_e \quad \text{where} \quad \langle \mathbf{P}^* | = (\mathbf{U}_D - \langle \mathbf{FP} \rangle)^{-1} \quad (3.20)$$

The way of characterizing an energy system, using as canonical variables the efficiency of its components and the distribution parameters is called " \mathcal{FP} representation". Thus, the universal formula of the efficiency of a system, no matter how complex, is:

$$\epsilon_T = \frac{{}^t\mathbf{y}_0 \langle \mathbf{P} | \mathbf{F}_e}{{}^t\mathbf{u} \mathbf{F}_e} \quad (3.21)$$

where ${}^t\mathbf{y}_0 = (y_{01}, \dots, y_{0n})$ is a vector that contains the distribution coefficients associated to the environment.

We are going to obtain the efficiency formulae of the gas turbine cycle. The distribution coefficient matrix is giving by:

$$\langle \mathbf{FP} \rangle = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & y_2 & 0 \\ y_3 & y_3 & 0 & 0 \\ 1 - y_3 & 1 - y_3 & 0 & 0 \end{bmatrix}$$

And the distribution coefficients with the environment:

$${}^t\mathbf{y}_0 = \begin{bmatrix} 0 & 0 & 1 - y_2 & 1 \end{bmatrix}$$

Applying Eqn (19), we can get the product of each component as a function of the distribution parameters and component efficiencies, using a symbolic computation application:

$$P_1 = \epsilon_1 E_1$$

$$P_2 = \frac{y_2 y_3 \epsilon_1 \epsilon_2 \epsilon_3}{1 - y_2 y_3 \epsilon_2 \epsilon_3} E_1$$

$$P_3 = \frac{y_2 y_3 \epsilon_1 \epsilon_3}{1 - y_2 y_3 \epsilon_2 \epsilon_3} E_1$$

$$P_4 = \frac{(1 - y_3) \epsilon_1 \epsilon_4}{1 - y_2 y_3 \epsilon_2 \epsilon_3} E_1$$

Then the efficiency of the total system is written as:

$$\epsilon_T = \frac{(1 - y_2) P_3 + P_4}{E_1} = \frac{(1 - y_2) y_3 \zeta_3 + (1 - y_3) \zeta_4}{1 - y_2 y_3 \zeta_2 \zeta_3 \zeta_1}$$

3.2.3 Irreversibility and Cost

The exergy cost of the product of each component of the plant can be expressed as a linear function of the external resources and the distribution coefficients:

$$P_i^* = \sum_{j=1}^n p_{ij}^*(y) E_{0j} \quad (3.22)$$

where p_{ij}^* are the elements of the operator $\langle \mathbf{P}^* |$. They are only functions of the distribution parameter and represent the amount of each external resources required to obtain one unit of the product on each component.

The linear operator associated to the exergy cost would be equal to the operator associated to the product, if the efficiency of each component will be equal to one. Thus, the exergy cost could be explained as the maximum amount of product that it is possible to obtain.

On the other hand, it is known that the exergy cost of a product is charged with both the exergy of product and all of the irreversibilities generated in the system to obtain it. This fact can be expressed using the relationship:

$$\mathbf{P}^* = \mathbf{P} + \langle \mathbf{P}^* | \mathbf{I} \quad (3.23)$$

The previous expression can be obtained from Eqns (19, 23):

$$\mathbf{P}^* = \langle \mathbf{P}^* | (\mathbf{K}_D - \langle \mathbf{FP} \rangle) \mathbf{P}$$

therefore:

$$\mathbf{P}^* = \langle \mathbf{P}^* | (\mathbf{K}_D - \mathbf{U}_D) \mathbf{P} + \langle \mathbf{P}^* | (\mathbf{U}_D - \langle \mathbf{FP} \rangle) \mathbf{P} = \langle \mathbf{P}^* | \mathbf{I} + \mathbf{P}$$

The elements of the matrix $\langle \mathbf{P}^* |$ show also the increase of the cost of the product due to the irreversibilities of the components where it is processed.

We are going to obtain the relationship between cost and irreversibilities for the gas turbine plant. The cost product operator, as a function of the distribution ratios, obtained from Eqn (23) is:

$$\langle \mathbf{P}^* | = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{y_2 y_3}{1 - y_2 y_3} & \frac{1}{1 - y_2 y_3} & \frac{y_2}{1 - y_2 y_3} & 0 \\ \frac{y_3}{1 - y_2 y_3} & \frac{y_3}{1 - y_2 y_3} & \frac{1}{1 - y_2 y_3} & 0 \\ \frac{1 - y_3}{1 - y_2 y_3} & \frac{1 - y_3}{1 - y_2 y_3} & \frac{y_2(1 - y_3)}{1 - y_2 y_3} & 1 \end{bmatrix}$$

Therefore, we can relate the cost of product, with the resources and the irreversibilities of each component:

$$P_1^* = F_1$$

$$P_2^* = F_2 + \frac{y_2}{1 - y_2 y_3} (y_3 I_1 + y_3 I_2 + I_3)$$

$$P_3^* = F_3 + \frac{y_3}{1 - y_2 y_3} (I_1 + I_2 + y_2 I_3)$$

$$P_4^* = F_4 + \frac{1 - y_3}{1 - y_2 y_3} (I_1 + I_2 + y_2 I_3)$$

And the numerical values of the product cost for design conditions, splinted by the contribution of the resources of the component and the irreversibilities of each component are shown in Table 2:

Table 3.2. Cost of product as irreversibility accumulation

	F	I_1	I_2	I_3	I_4	Total
P_1^*	11781	0	0	0	0	11781
P_2^*	2977	2662	197	252	0	6088
P_3^*	5783	4897	363	158	0	11201
P_4^*	3443	2915	216	94	0	6668

3.2.4 Exergoeconomic Cost

The fuel-product model has been applied to evaluate the exergy cost, but it can also be applied to the calculation of exergoeconomic costs.

The vector \mathbf{F}_e represents the exergy of the external resources, but also the exergy cost of the external resources. The meaning of this variable can be updated to include the investment and maintenance cost of the components of the system Z_i (\$/s) and the unitary economic cost of the external fuels c_i (\$/kJ), therefore the vector of economic cost of the external resources could be written as:

$$\mathbf{C}_e = [Z_i + c_i E_{0i}]_{i=1, \dots, n}$$

and the exergoeconomic cost of the product of each component is giving, in the FP representation, by:

$$\mathbf{C}_P = \langle \mathbf{P}^* | \mathbf{C}_e \quad (3.24)$$

$$\mathbf{C}_F = \mathbf{C}_P - \mathbf{Z} \quad (3.25)$$

Note that the operator of the exergy and exergoeconomic costs is the same, and only the costs of the external resources are modified.

3.3 The \mathcal{PF} Representation

In the previous section, we have studied the representation of the system's thermoeconomic variables as a function of its component efficiency, the distribution coefficients and the external resources.

The optimization and diagnosis analysis of a thermal system requires studying the behavior of the productive structure as a function of the total production objectives.

In this section, is presented an alternative representation that relates the Thermoeconomic variables of the system with the total plant product, the efficiency of its components, and a new type of parameter called *junction* ratio.

3.3.1 Junction Ratios

We define, in a similar way that the distribution coefficients, the *junction* coefficients r_{ij} , as the portion of the resources of the j -th component coming from the i -th

product.

$$r_{ij} = \frac{E_{ij}}{F_j} \quad (3.26)$$

that verifies the sum of all junction ratios of a component is equal one:

$$\sum_{i=0}^n r_{ij} = 1$$

Then, Eqn (2.1) could be written as:

$$P_i = E_{i0} + \sum_{j=1}^n r_{ij} F_j \quad i = 1, \dots, n \quad (3.27a)$$

$$F_T = \sum_{j=1}^n r_{0j} F_j \quad (3.27b)$$

It expresses the product of each components as a linear function of the fuel that its is transformed. The equation could be written in matrix from as:

$$\mathbf{P} = \mathbf{P}_s + \langle \mathbf{PF} \rangle \mathbf{F} \quad (3.28)$$

The vector \mathbf{P}_s ($n \times 1$) contains the exergy values of the final product obtained in each component, and $\langle \mathbf{PF} \rangle$ is a ($n \times n$) matrix which elements are the junction coefficients r_{ij}

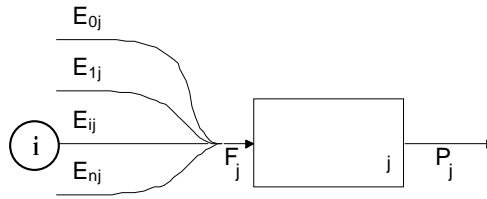


Figure 3.2. Definition of Junction Parameters

Thus, we can express the Thermoeconomic variables of a system, in a complementary way to FP representation, as a function of:

- The final product: \mathbf{P}_s
- The efficiency of each component: \mathbf{K}_D
- The junction components: $\langle \mathbf{PF} \rangle$.

This way of representing the thermoeconomic variables of a system is called \mathcal{PF} representation. The formulae of fuel, product and irreversibility in this representation are:

$$\mathbf{P} = |\mathbf{P}\rangle \mathbf{P}_s \quad \text{where} \quad |\mathbf{P}\rangle = (\mathbf{U}_D - \langle \mathbf{PF} \rangle \mathbf{K}_D)^{-1} \quad (3.29a)$$

$$\mathbf{F} = |\mathbf{F}\rangle \mathbf{P}_s \quad \text{where} \quad |\mathbf{F}\rangle = \mathbf{K}_D |\mathbf{P}\rangle \quad (3.29b)$$

$$\mathbf{I} = |\mathbf{I}\rangle \mathbf{P}_s \quad \text{where} \quad |\mathbf{I}\rangle = (\mathbf{K}_D - \mathbf{U}_D) |\mathbf{P}\rangle \quad (3.29c)$$

3.3.2 Unit Exergy Consumption

In order to simplify previous expressions, we could introduce the matrix:

$$\langle \mathbf{KP} \rangle \equiv \langle \mathbf{PF} \rangle \mathbf{K}_D \quad (3.30)$$

We denote its elements as κ_{ij} , which are defined as:

$$\kappa_{ij} = r_{ij}k_j = \frac{E_{ij}}{P_j} \quad (3.31)$$

They represent the amount of resources coming from the i -th component required to obtain a unit of the product of the j -th component, thus we call them, 'marginal exergy consumptions', and verify that their sum of components is equal to unit exergy consumption:

$$\sum_{i=0}^n \kappa_{ij} = k_j \quad (3.32)$$

The total resources of the system may be obtained as:

$$F_T = \sum_{j=1}^n \kappa_{0j}P_j \quad \text{or} \quad F_T = {}^t\kappa_e |\mathbf{P}\rangle \mathbf{P}_s \quad (3.33)$$

where ${}^t\kappa_e \equiv (\kappa_{01}, \dots, \kappa_{0n})$, is a $(n \times 1)$ vector whose elements contain the marginal exergy consumption of the system-input resources.

3.3.3 Cost Equations

If we substitute the marginal exergy consumption in the cost Eqn. (12), we get:

$$k_{P,i}^* = \kappa_{0i} + \sum_{j=1}^n \kappa_{ji}k_{P,j}^* \quad i = 1, \dots, n \quad (3.34)$$

This equation relates the unit exergy cost of the products with the marginal exergy consumption of each component, and the following holds:

$$\mathbf{k}_P^* = {}^t|\mathbf{P}\rangle \kappa_e \quad (3.35)$$

The cost of the fuel of each component can be obtained as:

$$k_{F,i}^* = r_{0i} + \sum_{j=1}^n r_{ji}k_{P,j}^* \quad (3.36)$$

The \mathcal{PF} representation formulae for the gas turbine cycle could be easily obtained. The unit consumption matrix is giving by:

$$\langle \mathbf{KP} \rangle = \begin{bmatrix} 0 & 0 & r_1k_3 & r_1k_4 \\ 0 & 0 & r_2k_3 & r_2k_4 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And the marginal exergy consumption coefficients related with the environment:

$${}^t\kappa_e = \begin{bmatrix} k_1 & 0 & 0 & 0 \end{bmatrix}$$

The product operator, obtained from Eqn (31), using the *Matlab* symbolic computation toolbox package is:

$$|\mathbf{P}\rangle = \begin{bmatrix} 1 & \frac{\kappa_{13}\kappa_{32}}{1 - \kappa_{23}\kappa_{32}} & \frac{\kappa_{13}}{1 - \kappa_{23}\kappa_{32}} & \frac{\kappa_{14}}{1 - \kappa_{23}\kappa_{32}} \\ 0 & \frac{1}{1 - \kappa_{23}\kappa_{32}} & \frac{\kappa_{23}}{1 - \kappa_{23}\kappa_{32}} & \frac{\kappa_{24}}{1 - \kappa_{23}\kappa_{32}} \\ 0 & \frac{\kappa_{32}}{1 - \kappa_{23}\kappa_{32}} & \frac{1}{1 - \kappa_{23}\kappa_{32}} & \frac{\kappa_{32}\kappa_{24}}{1 - \kappa_{23}\kappa_{32}} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, the total fuel of the system is written as:

$$F_T = \frac{\kappa_{01} (\kappa_{13}E_6 + \kappa_{14}E_7)}{1 - \kappa_{23}\kappa_{32}}$$

Analyzing these expressions and comparing with Figure 2 we can observed that sequential/parallel processes appear in the numerator and recycling productive processes appear in the denominator.

The numerical values of unit cost of product can be obtained solving the system of linear equation given in Eqn (38). The results are shown in Table 3.

Table 3.3. Thermoeconomic values for design conditions of TGAS

#	F (kW)	P (kW)	I (kW)	k	k_F^*	k_P^*	P* (kW)
1	11781	6631	5150.1	1.7767	1	1.7767	11781
2	2977	2595	382.2	1.1473	2.0451	2.3464	6088
3	5783	5477	306	1.0559	1.9369	2.0451	11201
4	3443	2355	1087.7	1.4619	1.9369	2.8315	6668

3.3.4 The \mathcal{FP} and \mathcal{PP} Relationships

Two different ways for the evaluation of the cost of the products, as a function of the efficiencies of the components, the external resources and the distribution or junction coefficients, have been explained: the FP and the PF representations.

The information flow in the FP representation has the direction of the productive process. If the external resources are known, it is possible to evaluate the rest of the thermoeconomic properties of the flows, including the exergy and the cost of the final products. The PF representation uses an equivalent logic structure, but the information has the opposite direction. If we know the plant product it is possible

to determine the resources required for obtaining any internal product, and its cost. Both representations are complementary and figure out the complete picture of the production process and cost formation.

These variables of both representations are related by the expression:

$$y_{ij}P_j = \kappa_{ji}P_i \quad (3.37)$$

Or in matrix form:

$$\langle \mathbf{FP} \rangle = \mathbf{P}_D^t \langle \mathbf{KP} \rangle \mathbf{P}_D^{-1} \quad (3.38)$$

It allows obtaining the operators of the FP representation from the PF operator and vice versa:

$$\langle \mathbf{P}^* | = \mathbf{P}_D^t | \mathbf{P} \rangle \mathbf{P}_D^{-1} \quad (3.39)$$

As an example of the use of the correspondence between both representations, the equivalent formula of the cost/irreversibility relationship is obtained:

Substituting the expression (eq x) in that equation, we get:

$$\mathbf{k}_p^* = \mathbf{u} + {}^t | \mathbf{P} \rangle \mathbf{P}_D^{-1} \mathbf{I} \quad (3.40)$$

On the other hand the irreversibility vector could be written as:

$$\mathbf{I} = \mathbf{P}_D (\mathbf{K}_D - \mathbf{U}_D) \mathbf{u}$$

Thus, substituting it in the previous equation, we get:

$$\mathbf{k}_p^* = \mathbf{u} + {}^t | \mathbf{I} \rangle \mathbf{u}$$

It could be written in scalar format as:

$$k_{p,i}^* = 1 + \sum_{j=1}^n \phi_{ji} \quad (3.41)$$

This relationship represents an alternative method for evaluating the unit exergy cost of the product, as sum of contribution of the component irreversibilities. The term ϕ_{ij} represents the irreversibility generated in the j -th component to obtain a unit of the i -th product.

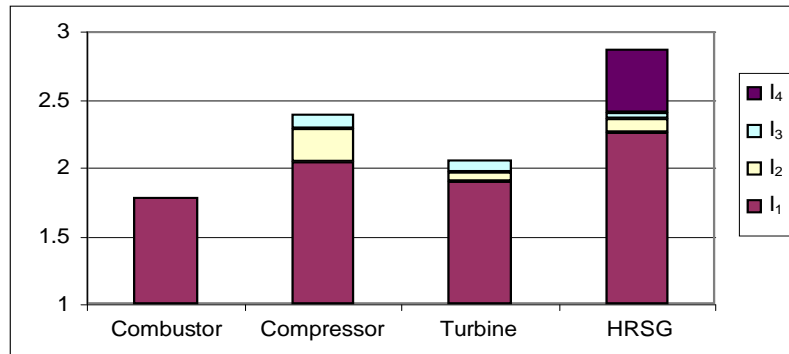


Figure 3.3. Exergy cost process formation

Figure 5 shows how the unit cost of product is obtained as the sum of the irreversibility contribution, according to Eqn (46), in a similar way as Table 2. The main contribution of the product cost is the irreversibility of the combustor.

3.4 Impact of resources consumption

As an application of the formulae obtained in previous sections, we will obtain the formulae to determine the impact on resources consumption, additional fuel plant consumption, when the efficiency or the unit exergy consumption of its components varies.

A performance test or a thermodynamic model simulator properly fitted to the real operation of the system provides the actual operating values of the plant. From these operation data the unit exergy consumptions and the thermoeconomic variables can be calculated for the operating conditions. These values can be compared with respect to those corresponding to the design values of the plant. Usually in operation, several plant devices suffer a degradation of their behavior with respect to the design conditions and, as a consequence, their efficiencies decrease and correspondingly their unit exergy consumptions increase.

The values of the unit exergy consumption increase are found as:

$$\Delta\kappa_{ij} = \kappa_{ij}(x) - \kappa_{ij}(x^0) \quad (3.42)$$

where x represents the actual values and x^0 a reference value.

From Eqn (37), it is possible to obtain the increment of the total resources of a plant in operation conditions regarding the reference conditions:

$$\Delta F_T = \Delta^t \kappa_e \mathbf{P}^0 + {}^t \kappa_e \Delta \mathbf{P} \quad (3.43)$$

The increase of the component production, may be expressed in terms of the unit exergy consumption as:

$$\Delta \mathbf{P} = \Delta \mathbf{P}_s + \Delta \langle \mathbf{KP} \rangle \mathbf{P}^0 + \langle \mathbf{KP} \rangle \Delta \mathbf{P} \quad (3.44)$$

therefore:

$$\Delta \mathbf{P} = |\mathbf{P}\rangle \left(\Delta \mathbf{P}_s + \Delta \langle \mathbf{KP} \rangle \mathbf{P}^0 \right) \quad (3.45)$$

The total fuel plant production is imposed by the external demand, in this case it is reasonable to analyze the fuel impact when there is no change in the total plant production, then Eqn x could be written as:

$$\Delta F_T = \left(\Delta^t \kappa_e + {}^t \mathbf{k}_p^* \Delta \langle \mathbf{KP} \rangle \right) \mathbf{P}^0 \quad (3.46a)$$

or in scalar format:

$$\Delta F_T = \sum_{i=1}^n \left(\sum_{j=0}^n k_{p,j}^* \Delta \kappa_{ji} \right) P_i^0 \quad (3.46b)$$

The above equation allows us to express the additional resource consumption ΔF_T also called *Fuel Impact*, as the sum of the contributions of each component malfunction.

A first conclusion from this result is that there is no equivalence between the irreversibilities of the components of an installation, the more advanced in the process the component location is, the bigger the increment on the resources consumption is, for the same amount of malfunction $\Delta \kappa_{ji} P_i^0$

3.5 Closure

Symbolic thermoeconomics has been presented as a methodology for the thermoeconomic analysis of energy systems. Its main objective is the study of the production process and cost-formation of energy systems.

The Exergy Cost Theory, defines a new thermodynamic function: the *exergy cost*, which values the real consumption of useful energy, needed to obtain the final products of the plant. *Symbolic Thermoeconomics*, use the premises of Exergy Cost Theory, to study the structure of thermal systems animated by a productive purpose. A set of subsystem properties like the efficiency, the junction and distribution parameters have been defined, and they constitute the basic or canonical variables of two structural representations of a system the FP and the PF representation. The F_e , P_s variables relate the system with its environment, while k , y , r are variables associated with its internal structure. This system characterization permits the definition of functions of the thermoeconomic properties, like exergy and cost.

The main objective of Symbolic Thermoeconomics consists in explaining the cost formation process, providing a set of numerical procedures as well as general formulae valid for any state of the system and which solely depends on the productive structure and its interaction with the environment. In this way, the cost of the product of a generic component of the system could be expressed as the amount of external resources required for obtaining it, (Eqn 23) or as the sum of the internal resources plus the sum all the irreversibilities generated across its process formation (Eqn 26).

Symbolic Thermoeconomics is not an alternative method to conventional optimization or simulation techniques of actual complex systems, but complements them. It gives a set of tools, designed to be implemented into computer software applications, which allow to solve problems in different thermoeconomic fields like cost accounting, diagnosis and optimization.

Structural Theory of Thermoeconomics

Characteristic equations and average and marginal costs are analyzed in this article. As a consequence the structural theory is a general mathematical formalism either for thermoeconomic cost accounting and/or optimization methods, providing a common basis of comparison among the different thermoeconomic methodologies, which could be considered the standard formalism for thermoeconomics.

During the three decades from 1972 to 2002 various thermoeconomic methodologies have been developed. All of them have in common cost calculated on a rational basis, which is the Second Law of Thermodynamics. This cost is a very useful tool for solving problems in complex energy systems, such as rational price assessment of the products of a plant based on physical criteria, local optimization, or operation diagnosis. These problems are difficult to solve using conventional energy analysis techniques based on the First Law of Thermodynamics.

There are two main groups of thermoeconomic methods: a) cost accounting methods, which use average costs as a basis for a rational price assessment, and b) optimization methods, which employ marginal costs in order to minimize the costs of the products of a system or a component.

When comparing different thermoeconomic methodologies and the underlying ideas of their models, the reader is faced with as many nomenclatures, concepts, and names as there are existing methods. This could be one of the factors that impede a faster development of thermoeconomics. In order to avoid unnecessary confusion and provide a common basis for comparing different thermoeconomic methods, a common mathematical language for thermoeconomics is essential.

Such a common mathematical formulation is provided by the structural theory of thermoeconomics, which can reproduce the results of any thermoeconomic methodology, both cost accounting and optimization, employing a linear thermoeconomic model. The most systematic and widespread thermoeconomic methodologies developed until now use linear models, or models that are easy to linealize. For this reason the structural theory provides a common mathematical formulation for the different thermoeconomic methodologies.

In this article, the concept of cost is analyzed in detail and the mathematical formalism of the structural theory is presented. This theory allows:

- Analysis of the process of cost formation, providing a physical and mathematical interpretation to the rules of cost apportioning
- Unification of the concepts of cost provided by the different cost accounting methodologies and some of the optimization methodologies.

4.1 Marginal Cost and Characteristic Equations

When performing a cost analysis in a system we can distinguish between *average costs*, which are ratios and express the average amount of resources per unit of product, and *marginal costs*, which are partial derivatives and indicate the additional resources required to generate one more unit of the product under specified conditions

In this section the concept of marginal costs in thermoeconomics is analyzed in detail, and some of its applications are presented. In order to help the reader follow the proposed arguments, the same cogeneration plant presented in other articles within this theme will be used as an example application (see Figure 1.1)

4.1.1 Characteristic Equations

The thermoeconomic model that is the mathematical representation of the productive structure consists of a set of mathematical functions called *characteristic equations*. They express each inlet flow as a mathematical function of the outlet flows, for all the productive structure process units and a set of internal parameters \mathbf{x} :

$$E_i = g_i(\mathbf{x}, \mathbf{E}_j) \quad i \in \mathcal{E}_u \quad j \in \mathcal{S}_u \quad u = 1, \dots, n \quad (4.1)$$

where the index i refers to the input flows of the process unit l , the index j refers to the output flows of the process unit l , and m and s are respectively the number of flows and the number of system outputs considered in the productive structure. Every flow is an input flow of a process unit, and an output flow of another process unit or to the environment. For the flows interacting with the environment, we define:

$$E_i = \omega_i(\mathbf{x}) \quad i \in \mathcal{E}_0 \quad (4.2)$$

where ω_i is the total system product, that is, an function that determines the total plant product from the internal parameters \mathbf{x} . The characteristic equations for the cogeneration plant of our example (see Figure 1.1) are shown in Table 1.

Table 4.1. Characteristic equations of the cogeneration plant

Nr	Component	Entry	Outlet	Equation
1	Combustor	[1,2]	[3]	$E_1 = g_1(x, E_3)$ $E_2 = g_2(x, E_3)$
2	Turbine	[3]	[4,7, 8]	$E_3 = g_3(x, E_4, E_7, E_8)$
3	Compressor	[7]	[2]	$E_5 = g_5(x, E_2)$
4	H.R.S.G.	[4]	[6]	$E_4 = g_4(x, E_6)$
0	Environment	[6, 8]	[1]	$E_6 = \omega_6(x)$ $E_8 = \omega_8(x)$

The inlet and outlet flows of the productive structure units are of extensive magnitude, and are the product of a quantity (usually mass flow rate) and a quality (specific magnitude). The magnitudes applied by most thermoeconomic methodologies are exergy, negentropy, and money. Other magnitudes, like enthalpy or entropy, can also be used.

The internal variables appearing in the thermoeconomic model depend on the behavior of the subsystem, and they are presumably independent of mass flow rates. This implies that relations like efficiencies or pressure and temperature ratios—which are mainly independent of the quantity of the exiting flows—can be used as internal parameters.

Note that the main objective of the productive structure, and hence of the thermoeconomic model, consists of sorting the thermodynamic magnitudes related to the physical mass and energy flow-streams connecting the plant subsystems in a different way from the equations modeling the physical plant behavior, in order to determine explicitly for each subsystem its energy conversion efficiency.

It is important to keep in mind that thermoeconomics connects thermodynamics with economics. That is, by sorting the thermodynamic properties of the physical mass and energy flow-streams of a plant, which in turn provide the energy conversion efficiency of each subsystem, thermoeconomics analyzes the degradation process of energy quality through an installation.

Depending on the scope of the analysis, a subsystem can be identified as a separate piece of equipment, a part of a device, several process units, or even the whole plant. Sometimes the objective consists of analyzing a plant in great detail. In this case it is advisable, if possible, to identify each subsystem with a separate physical process (heat transfer, pressure increase or decrease, and chemical mixture or reaction) in order to locate and quantify, separately if possible, each thermal, mechanical, and chemical irreversible process occurring in the plant. If the objective consists of analyzing a macrosystem composed of several plants, in this case the more convenient approach would probably be to consider each separate plant as a subsystem.

Thus, thermoeconomics always performs a systemic analysis, no matter how complex the system, oriented towards locating and quantifying the energy conversion efficiency and the process of energy quality degradation. It is not within the scope of thermoeconomics to model the behavior of the process units, which is done by the mathematical equations of the physical model.

Even though it is not the objective of thermoeconomics to simulate the behavior of the subsystems, it is very important to build a thermoeconomic model with physical meaning. This is the reason, as already explained, for defining different thermoeconomic models for the same plant. Depending on the aggregation level and the nature of the thermoeconomic equations, the model will contain physical information about the actual system behavior with different degrees of detail. The results obtained from a very rough thermoeconomic model, which is not sensitive to any physical detail related to the actual behavior of the plant, will probably be useless.

4.1.2 General Equation of Marginal Cost

Once the thermoeconomic model has been defined and the characteristic equations corresponding to the productive structure of the system are known, the costs of all flows in the productive structure can easily be calculated.

The thermoeconomic model (characteristic equations) of an energy system contains the mathematical dependence between the resources consumed and plant flows (products and internal flows). Each flow, as a process unit input, is a function (defined by its characteristic equation) of a set of internal variables, x , and the output flows of the process unit. The cost of plant resources can be then expressed

by the following equation:

$$E_0 = \sum_{i=1}^e c_{0,i} E_i \quad (4.3)$$

When the variation of the resources consumed in the plant concerning a flow is calculated, the chain rule can be applied:

$$\begin{aligned} \frac{\partial E_0}{\partial E_i} &= c_{0,i} \quad j \in \mathcal{S}_0 \\ \frac{\partial E_0}{\partial E_i} &= \sum_{\substack{j=1 \\ j \neq i}}^m \frac{\partial E_0}{\partial E_j} \frac{\partial g_j}{\partial E_i} \quad j \in \mathcal{S}_u \quad u = 1, \dots, n \end{aligned} \quad (4.4)$$

The equation (3.4) represents the marginal cost, which evaluates the additional consumption of resources when an additional unit of the flow $-i-$ is produced, under the conditions that the internal variables, x , do not vary throughout the process. We can denote these marginal costs as k_i^* , and:

$$\kappa_{ji} = \frac{\partial g_j}{\partial E_i}$$

as the marginal consumption of flow $-j-$ to produce the flow $-i-$. With these, we can rewrite the previous expressions as:

$$\begin{aligned} k_i^* &= c_{0,i} \quad j \in \mathcal{S}_0 \\ k_i^* &= \sum_{\substack{j=1 \\ j \neq i}}^m k_j^* \kappa_{ji} \quad j \in \mathcal{S}_u \quad u = 1, \dots, n \end{aligned} \quad (4.5)$$

These equations can be written in matrix notation as follows:

$$(\mathbf{U}_D - {}^t\langle \mathbf{G} \rangle) \mathbf{k}^* = \mathbf{c}_e \quad (4.6)$$

where \mathbf{U}_D is the identity matrix ($m \times m$), $\langle \mathbf{G} \rangle$ is a ($m \times m$) matrix containing the Jacobian of the characteristic equations, its elements are the marginal exergy consumptions κ_{ij} , and \mathbf{k}^* is a vector ($m \times 1$) which contains the marginal costs of each flow

Note that when the boundary of the system analyzed coincides with the limits of the plant studied, then the unit exergy cost of each fuel entering the plant is considered equal to 1 because there is no energy quality degradation or exergy destruction at the very beginning of the productive process. Hence, the amount of exergy consumed to obtain each plant's fuel is its own exergy content, and therefore its unit exergy cost is equal to 1.

Then, if the characteristic equations and the marginal consumptions for each process unit are known, the marginal cost k^* for each flow can be obtained by solving the system of linear Eqs. (6).

If the unit costs of the inlet plant fuels and the characteristic equations are known, the previous equations are a set of m equations with m unknowns, which are the marginal costs. Note that this set of equations shows the process of cost formation on the productive structure: that is, how the cost is generated through the process units of the plant.

Note that the proposed procedure to calculate the marginal cost of all the flows of a plant is general and valid for any thermoeconomic formulation that uses characteristic equations connecting inlet and outlet flows of each process unit.

The thermoeconomic cost calculation procedure considering monetary units is similar to that explained in the previous paragraphs, but in this case the input plant resources are expressed in monetary units and the process unit capital cost Z must be taken into account. Thus, the capital cost of each process unit Z can be considered an external flow of the plant resources from the environment to the process unit (see Figure 1). This will represent the monetary units per second needed to compensate for the depreciation, maintenance cost, and so on of the process unit.

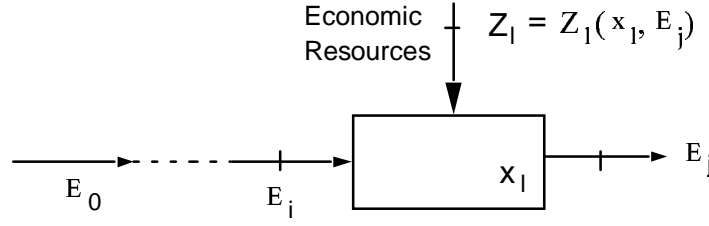


Figure 4.1. Economic resources scheme

According to marginal cost analysis, Z represents an environmental resource and can be handled in the same mathematical way as energy resources. The amount of resources consumed when manufacturing a device is, in fact, the amount of resources consumed to obtain the plant products. Then the marginal unit cost $\partial Z/\partial E$ can be considered as a resources cost and include in c_e .

In monetary units the equation (3.4), can be rewritten as follows:

$$\begin{aligned} \frac{\partial C_0}{\partial E_i} &= c_{0,i} \quad j \in S_0 \\ \frac{\partial C_0}{\partial E_i} &= \frac{\partial Z_u}{\partial E_i} + \sum_{\substack{j=1 \\ j \neq i}}^m \frac{\partial C_0}{\partial E_j} \kappa_{ji} \quad j \in S_u \quad u = 1, \dots, n \end{aligned} \quad (4.7)$$

where C_0 represents the resources consumed in the whole system but in this case expressed in monetary units, an Z_u is the capital cost of the device u . Note that in this case the unit cost is expressed in monetary units per unit of exergy.

For the example of the co-generation plant, eqs. (6a) and (6b) can be written as:

$$\begin{aligned} k_1^* &= c_1 & k_2^* - \kappa_{72}k_7^* &= \frac{\partial Z_2}{\partial E_2} \\ k_3^* - \kappa_{13}k_1^* - \kappa_{23}k_2^* &= \frac{\partial Z_1}{\partial E_3} & k_4^* - \kappa_{34}k_3^* &= \frac{\partial Z_3}{\partial E_4} \\ k_5^* - \kappa_{45}k_4^* &= \frac{\partial Z_4}{\partial E_5} & k_6^* - \kappa_{46}k_4^* &= \frac{\partial Z_4}{\partial E_6} \\ k_7^* - \kappa_{37}k_3^* &= \frac{\partial Z_3}{\partial E_7} & k_8^* - \kappa_{38}k_3^* &= \frac{\partial Z_3}{\partial E_8} \end{aligned}$$

4.1.3 Generalized Fuel Impact

Just as k^* was defined as a marginal cost when production is modified, we can also obtain the marginal cost when the internal variables x are modified. The plant resources can be expressed as a function of internal parameters of the devices:

$$C_0(\mathbf{x}) = {}^t\mathbf{c}_e \mathbf{E}(\mathbf{x}) + {}^t\mathbf{u} \mathbf{Z}(\mathbf{x}, \mathbf{E}(\mathbf{x})) \quad (4.8)$$

Similarly, applying the chain rule, we get:

$$\frac{\partial C_0}{\partial x} = \left({}^t\mathbf{c}_e + {}^t\mathbf{u} \frac{\partial \mathbf{Z}}{\partial \mathbf{E}} \right) \frac{\partial \mathbf{E}}{\partial x} + {}^t\mathbf{u} \frac{\partial \mathbf{Z}}{\partial x} \quad (4.9)$$

which can also be expressed as follows:

$$\frac{\partial C_0}{\partial x} = {}^t\mathbf{k}^* \frac{\partial \mathbf{g}}{\partial x} + {}^t\mathbf{u} \frac{\partial \mathbf{Z}}{\partial x} \quad (4.10)$$

This equation is the generalization *fuel impact* formula presented in previous chapter, and expresses the effect on additional resource consumption when an internal parameter x_i is modified. It is the basis for the thermoeconomic diagnosis and optimization explained in detail next chapters.

4.1.4 Lagrange Multipliers and Marginal Costs

The most developed thermoeconomic optimization methodologies, as *Thermoeconomic Functional Analysis* use the Lagrange multipliers optimization method to calculate the marginal costs defined in the previous section. In this section it is proved that the Lagrange multipliers are the marginal costs defined in Eq. (2), or:

$$\lambda_i = \frac{\partial E_0}{\partial E_i} \quad i = 1, \dots, m$$

This multiplier represents the variation of the objective function C_0 , the plant resources, concerning the state variable E_i , the exergy of a flow stream, which is the same definition as that presented in Eq(32).

The main purpose of an optimization problem consists of finding the values of a set of design variables, \mathbf{x} , that minimize the cost of consumed resources needed to obtain a product:

$$C_0(\mathbf{x}, \mathbf{E}) = \sum_{i \in S_0} c_i E_i + \sum_{u \in N} Z_u(\mathbf{x}, \mathbf{E}) \quad (4.11)$$

constrained by the conditions imposed by the characteristic equations and the total plant production objective:

$$\begin{aligned} E_i &= g_i(\mathbf{x}, \mathbf{E}) & i \in \mathcal{E}_u & \quad u = 1, \dots, n \\ E_i &= \omega_i(\mathbf{x}) & i \in \mathcal{E}_0 \end{aligned} \quad (4.12)$$

The Lagrange theorem establishes that the optimum of the objective function, Eq. (15), constrained by the restrictions, Eq. (16), is the same as the objective function without constraints, or the Lagrange function:

$$\begin{aligned} L(\mathbf{x}, \mathbf{E}, \Lambda) &= \sum_{i \in S_0} c_i E_i + \sum_{u \in N} Z_u(\mathbf{x}, \mathbf{E}) \\ &+ \sum_{\substack{u \in N \\ i \in \mathcal{E}_u}} (\lambda_i (g_i(\mathbf{x}, \mathbf{E}) - E_i)) + \sum_{i \in S_0} \lambda_i (\omega_i - E_i) \end{aligned} \quad (4.13)$$

This Lagrangian function can also be written as follow:

$$\begin{aligned}
 L(\mathbf{x}, \mathbf{E}, \Lambda) = & \sum_{i \in S_0} (c_i - \lambda_i) E_i \\
 & + \sum_{u \in N} \left(\Gamma_u(\mathbf{x}, \mathbf{E}, \Lambda) - \sum_{i \in S_u} \lambda_i E_i \right) + \sum_{i \in S_0} (\lambda_i \omega_i)
 \end{aligned} \tag{4.14}$$

where:

$$\Gamma_u(\Lambda, \mathbf{x}, \mathbf{E}) = Z_u(\mathbf{x}, \mathbf{E}) + \sum_{i \in E_u} \lambda_i g_i(\mathbf{x}, \mathbf{E}) \tag{4.15}$$

represents the cost of the inlet resources to the u component. The values of the Lagrange multipliers, λ_i , fulfill the equations:

$$\begin{aligned}
 \lambda_i &= c_i \quad i \in S_0 \\
 \lambda_i &= \frac{\partial \Gamma_u}{\partial E_i} = \frac{\partial Z_u}{\partial E_i} + \sum_{j \in E_u} \lambda_j \frac{\partial g_j}{\partial E_i} \quad i \in S_u \quad u = 1, \dots, n
 \end{aligned} \tag{4.16}$$

These relationships are valid for any feasible operating point fulfilling the constraints. Note that these equations are the same as those defined by Eq. (9), which is the marginal cost.

4.2 Structural Theory of Thermoeconomics

The general model of marginal costs explained in previous section provides a formal and general way of cost calculation, but leads to two important remarks that should be analyzed carefully.

First, as already mentioned, the most widespread thermoeconomic methods use linear thermoeconomic models, or models that are easy to linearize.

On the other hand, the proposed method of cost calculation provides costs that can be used in optimization, because the costs obtained are marginal costs, as was proved in the previous section. However, what is the relationship between the marginal costs and the average costs provided by cost accounting methods? Can the general model of marginal costs be used for cost accounting purposes?

The answer to these questions is presented in the next sections.

4.2.1 Linear Model of Characteristic Equations

Let us consider the characteristic equations as linear functions with respect to the magnitude that represents the flow streams:

$$g_i(\mathbf{x}, \mathbf{E}) = \sum_{j \in S_u} \alpha_{ij}(\mathbf{x}) E_j \quad i \in E_u \tag{4.17}$$

The outlet exergy flows here are independent variables with respect to the internal parameters. If this condition is achieved, then the marginal consumption can be selected as the coefficients of the characteristic equation, that is:

$$\alpha_{ij}(\mathbf{x}) = \kappa_{ij}(\mathbf{x}) = \frac{\partial g_i}{\partial E_j}(\mathbf{x}) \quad (4.18)$$

The cost of the external resources of a system, can be written as:

$$C_0(\mathbf{x}, \mathbf{E}) = {}^t\mathbf{c}_e \mathbf{E} + {}^t\mathbf{u} Z(\mathbf{x}, \mathbf{E}) \quad (4.19)$$

constrained by the characteristic equations:

$$E_i = \sum_{j=1}^n \kappa_{ij}(\mathbf{x}) E_j \quad i \in \mathcal{E}_u \quad u = 1, \dots, n \quad (4.20)$$

$$E_i = \omega_i(\mathbf{x}) \quad i \in \mathcal{E}_0$$

In matrix notation, these equations can be written in a compact form:

$$(\mathbf{U}_D - \langle \mathbf{G} \rangle) \mathbf{E} = \mathbf{E}_s \quad (4.21)$$

\mathbf{E}_s is a $(m \times 1)$ vector containing the values of external variables (plant products), ω_i .

Note that the matrix containing the system of equations needed for the calculation of marginal costs (see Eq.3.6) is the transpose matrix defining the characteristic equations (Eq 3.20). This fact allows one to obtain the system of cost equations directly once the characteristic equations have been defined. In linear algebra these structures are called respectively primal and dual.

4.2.2 Average and Marginal Costs

In this subsection it is proved that when the characteristic equations of the system are linear functions concerning the extensive magnitude E , the systems of equations required for the calculation of marginal and average costs coincide. That is, when the characteristic equations are linear functions, the marginal and average costs are calculated using the same mathematical procedure, and therefore they coincide. This is a very important result because the marginal and average costs can be calculated using the same procedure. To illustrate this fact, let us consider a generic

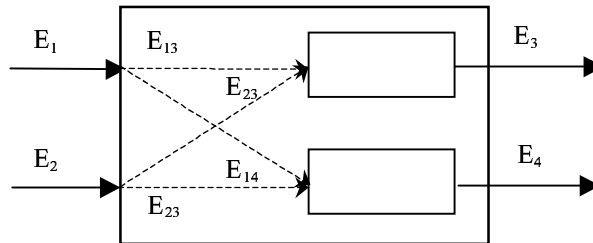


Figure 4.2. General component scheme

component or subsystem of a plant with several inlet and outlet flows. For the sake of simplicity we shall consider a general subsystem with two inlet flows and two

outlet flows, depicted in Figure 3.2 The characteristic equations that describe the behavior of this component are:

$$E_i = \sum_{j \in S_u} \kappa_{ij} E_j \quad i \in E_u \quad (4.22)$$

According to figure 3.2 the input resources could be decomposed as:

$$\begin{aligned} E_1 &= \kappa_{13} E_3 + \kappa_{14} E_4 \\ E_2 &= \kappa_{23} E_3 + \kappa_{24} E_4 \end{aligned} \quad (4.23)$$

These equations are concerned with the amount of inlet resources (E_1, E_2) consumed to obtain each one of the outlet flows (E_3, E_4). Correspondingly, the total amount of resources consumed for the whole system for obtaining E_3 and E_4 is:

$$\begin{aligned} \bar{k}_1^* E_1 &= \bar{k}_1^* \kappa_{13} E_3 + \bar{k}_1^* \kappa_{14} E_4 \\ \bar{k}_2^* E_2 &= \bar{k}_2^* \kappa_{23} E_3 + \bar{k}_2^* \kappa_{24} E_4 \end{aligned} \quad (4.24)$$

If the component is considered to be made up of two subsystems, then the equations modeling each subsystem are:

$$E_{13} = \kappa_{13} E_3 \quad (4.25a)$$

$$E_{14} = \kappa_{14} E_4 \quad (4.25b)$$

$$E_{23} = \kappa_{23} E_3 \quad (4.25c)$$

$$E_{24} = \kappa_{24} E_4 \quad (4.25d)$$

Eqs. (26a) and (27a) represent the resources needed to produce E_3 , and Eqs. (26b) and (27b) are the resources consumed to produce E_4 . Therefore, the total amount of resources consumed for the whole system required to obtain E_3 and E_4 are:

$$\begin{aligned} \bar{k}_1^* E_{13} + \bar{k}_2^* E_{23} &= \bar{k}_1^* \kappa_{13} E_3 + \bar{k}_2^* \kappa_{23} E_3 \\ \bar{k}_1^* E_{14} + \bar{k}_2^* E_{24} &= \bar{k}_1^* \kappa_{14} E_4 + \bar{k}_2^* \kappa_{24} E_4 \end{aligned} \quad (4.26)$$

The average cost of the outlet flows is:

$$\begin{aligned} \bar{k}_3^* &= \frac{\bar{k}_1^* \kappa_{13} E_3 + \bar{k}_2^* \kappa_{23} E_3}{E_3} = \bar{k}_1^* \kappa_{13} + \bar{k}_2^* \kappa_{23} \\ \bar{k}_4^* &= \frac{\bar{k}_1^* \kappa_{14} E_4 + \bar{k}_2^* \kappa_{24} E_4}{E_4} = \bar{k}_1^* \kappa_{14} + \bar{k}_2^* \kappa_{24} \end{aligned} \quad (4.27)$$

And the marginal costs of the outlet flows are:

$$\begin{aligned} k_3^* &= \frac{\partial C_0}{\partial E_3} = \frac{\partial C_0}{\partial E_1} \frac{\partial E_1}{\partial E_3} + \frac{\partial C_0}{\partial E_2} \frac{\partial E_2}{\partial E_3} = k_1^* \kappa_{13} + k_2^* \kappa_{23} \\ k_4^* &= \frac{\partial C_0}{\partial E_4} = \frac{\partial C_0}{\partial E_1} \frac{\partial E_1}{\partial E_4} + \frac{\partial C_0}{\partial E_2} \frac{\partial E_2}{\partial E_4} = k_1^* \kappa_{14} + k_2^* \kappa_{24} \end{aligned} \quad (4.28)$$

So the average and marginal costs of the flows E_3 and E_4 coincide. Both kinds of cost are the same because the equations modeling the component are linear functions with respect to the extensive magnitudes that characterize the outlet flows.

The general mathematical formulation of the cost generated in a component is the same for each cost and is not dependent on the position in the productive process. For this reason the result obtained is general.

4.3 Structural Theory as Standard for Thermoeconomics

As has been proved in this article, the structural theory of thermoeconomics provides a mathematical formalism that allows the unification of cost accounting and optimization theories in a common mathematical formulation, provided that linear mathematical functions connect the inputs and outputs of the devices.

The most important advantage of this unification is that variables and costs with different conceptual significance can be compared and better understood. As a result of this integration of different approaches, it has been possible to develop some useful thermoeconomic applications, such as energy audits, operation diagnosis, and local optimization, using the same mathematical formalism.

4.3.1 Structural Theory and Exergy Cost Theory

It has been proved that the marginal costs of the structural theory coincide with average costs when the characteristic equations are linear with respect to the extensive magnitude characterizing the outlet flows.

Now we are going to show that the cost assessment rules of the exergy cost theory can be obtained as a result of the structural theory. In other words, by applying directly the mathematical rules of.

At this point it is important to remark that the costs obtained applying the mathematical formalism of the structural theory, when the characteristic equations are linear functions with respect to the exergy of the outlet flows, are also a conservative property. Remember that average costs obtained applying cost accounting methods, as for instance, the exergy cost theory, are also a conservative property. Thus, using the structural theory notation, it can be written that:

$$\sum_{i \in E_u} k_i^* E_i = \sum_{j \in S_u} k_j^* E_j \quad (4.29)$$

This proposition is fulfilled by the structural theory when the characteristic equations are linear functions with respect to the exergy of the outlet flows:

$$\begin{aligned} \sum_{i \in E_u} k_i^* E_i &= \sum_{i \in E_u} k_i^* \left(\sum_{j \in S_u} \kappa_{ij} E_j \right) \\ &= \sum_{j \in S_u} \left(\sum_{i \in E_u} k_i^* \kappa_{ij} \right) E_j = \sum_{j \in S_u} k_j^* E_j \end{aligned} \quad (4.30)$$

The propositions of the Exergetic Cost Theory can be explained by the structural theory. It is a particular case, when characteristic equations are linear function, and uses average consumption instead marginal consumption.

P1 rule: It is an external assignation and applies in all cases.

P2 rule: It means the costs is a conservative property and it has been proved above in equation (32).

P3 rule: In a product bifurcation, the unit cost of products are equal. Let us to consider a generic bifurcation, as it is shown in fig 3.3, therefore from efficiency definition we get the characteristic equation:

$$E_i = \kappa E_j + \kappa E_h \quad (4.31)$$

and the dual equations for unit costs are:

$$k_j^* = k_h^* = \kappa k_i^* \quad (4.32)$$

thus proposition P3 is satisfied

P4 rule: If we have a continuous fuel stream, the unit cost of the output is equal to the unit cost of input flow. Now we consider E_j as an exhausted fuel, then from efficiency definition we get the characteristic equation as:

$$E_i = E_j + \kappa E_h \quad (4.33)$$

and the dual equations for unit costs are:

$$k_j^* = k_i^* \quad k_h^* = \kappa k_i^* \quad (4.34)$$

and also proposition P4 is satisfied.

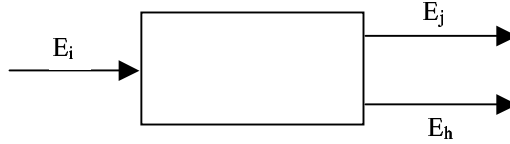


Figure 4.3. Component with product bifurcation

The characteristic equations provide a rational way for a general and objective assessment of cost in bifurcations based on the values of the marginal consumptions of local resources.

From now, cost allocation will not be a separated task from the assessment of the units behavior. In fact they constitute two side of a united activity. An specific cost assignment means that we are assuming an implicit behavior of the plant and vice versa.

In the case of the exergetic cost theory is the fuel – product definition or the exergetic efficiency which determine the characteristic equations and then the costs.

On another way structural theory provides an alternative, more efficient in computational terms to compute exergetic and/or exergoeconomic cost, as it is shown in the next example

We will to compute the unit exergetic cost of the flow stream of the cogeneration plant defined in figure 1.1. First we will determine the characteristic equations:

$$\begin{aligned} E_1 &= r_1 k_1 E_3 & E_2 &= r_2 E_3 \\ E_3 &= E_4 + k_3 (E_7 + E_8) & E_4 &= E_5 + k_4 E_3 \\ E_5 &= \omega_5 & E_6 &= \omega_6 \\ E_7 &= k_2 E_2 & E_8 &= \omega_8 \end{aligned}$$

Then the following system of linear equations could be used to determine the unit

exergetic cost of the flow, in an equivalent way that we use in chapter1:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -k_2 & 0 \\ -r_1 k_1 & -r_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 & 1 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -k_3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1^* \\ k_2^* \\ k_3^* \\ k_4^* \\ k_5^* \\ k_6^* \\ k_7^* \\ k_8^* \end{bmatrix} = \begin{bmatrix} c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4.3.2 General rules for cost assessments

Cost accounting methodologies determine the unitary cost of the plant products is based in the cost balance equation (31). In the hypothetic situation, that the result of its activity will be only a flow, this equation is enough to determine its unit costs, but in real plants is common that several products will be produced at the same time.

If we consider a generic component as it is shown in figure 3, its cost balance is written as:

$$B_i k_i^* = B_j k_j^* + B_h k_h^* \quad (4.35)$$

The exergy of the flows and the unit cost of the input fuel is known, therefore we have one equation and two unknowns, the unit cost of the outputs.

From the point of view of the structural theory, the characteristic, equation of this component is:

$$B_i = \kappa_{ij} B_j + \kappa_{ih} B_h \quad (4.36)$$

and the unit cost are determined by:

$$k_j^* = \kappa_{ij} k_i^* \quad k_h^* = \kappa_{ih} k_i^* \quad (4.37)$$

In order to compute the unit cost of the flows, we need to determine its unit consumptions, again we have one equation and two unknowns. We have seen that exergy cost theory introduce one additional equation, the unit cost of the product are equal : $k_j^* = k_h^*$, this condition expressed in terms of unit consumption is equivalent to $\kappa_{ih} = \kappa_{ij}$, it means that the amount of resources to obtain one unit of each product is the same. As we are exposed during the course this rule can not be used in a indiscriminate way, and requires the study of the conditions it could be applied to real systems.

According with this idea we could define a *combined distribution* parameter as:

$$\pi_i(\mathbf{x}) = \frac{\kappa_{ij}(\mathbf{x})}{\kappa_{ih}(\mathbf{x})} \quad (4.38)$$

Therefore, this parameter could be used as additional equation to determine the unit cost of the products, if we know the value of this parameter, the unit consumption of each product could be obtained combined with characteristic equation, and then the unit cost of the flows. Note that in the exergy cost theory the combined distribution parameter is the simplest one choice: $\pi(\mathbf{x}) = 1$.

4.3.3 Structural Theory and the Fuel–Product Model

The characteristic equations do not limit or impose any constraint on the productive model. In the case of the exergy cost theory it is defined through the fuel–product model described in the previous chapter. In a fuel–product diagram (see figure 1.2), the flows represent the fuels and products of the devices, as characterized by the outlet flows, which are the products, and the inlet flows, which are the fuels.

Hence, the characteristic equations corresponding to the fuel–product model are:

$$\begin{aligned} F_i &= k_i P_i \\ P_i &= E_{0i} + \sum_{j=1}^n r_{ij} F_j \end{aligned} \quad (4.39)$$

These two equations can be combined obtaining the characteristic equation of the PF representation:

$$P_i = E_{0i} + \sum_{j=1}^n \kappa_{ij} P_j \quad (4.40)$$

Thus, the equation of the resources of the whole system can be written in the case of the thermoeconomic model of the exergy cost theory as follows:

$$C_0 \equiv {}^t \kappa_e \mathbf{P} + {}^t \mathbf{u} \mathbf{Z} \quad (4.41)$$

which is constrained by the equations:

$$\mathbf{P} = \mathbf{P}_s + \langle \mathbf{KP} \rangle \mathbf{P} \quad (4.42)$$

Note that in the case of the fuel–product model, the matrix corresponding to the marginal exergy consumptions, $\langle \mathbf{G} \rangle$, is the $\langle \mathbf{KP} \rangle$ matrix.

Applying the definition of marginal cost to Eq (42,43), it holds:

$$\left(\mathbf{U}_D - {}^t \langle \mathbf{KP} \rangle \right) \frac{\partial C_0}{\partial \mathbf{P}} = \mathbf{z}_e \quad (4.43)$$

where \mathbf{z}_e is a $(n \times 1)$ vector containing the marginal costs of the resources entering the system:

$$\mathbf{z}_e = \kappa_e + \frac{\partial \mathbf{Z}}{\partial \mathbf{P}} \mathbf{u} \quad (4.44)$$

If the resources corresponding to the devices are proportional to their production, in other words, $Z_i = z_{p,i} P_i$ then Eq. (42) can be written as follows:

$$\mathbf{z}_e = \kappa_e + \mathbf{z}_p \quad (4.45)$$

And the equation of the total resources entering the system can be written in its dual form as follows:

$$C_0 \equiv {}^t \mathbf{k}_p^* \mathbf{P}_s \quad (4.46)$$

constrained by the conditions:

$$\mathbf{k}_p^* = \mathbf{z}_e + {}^t \langle \mathbf{KP} \rangle \mathbf{k}_p^* \quad (4.47)$$

which coincide with the equations of the unit exergy costs of the PF representation

An expression similar to the *generalized fuel impact* can be obtained by applying Eq. (9) to the characteristic equations of the fuel–product model:

$$\frac{\partial C_0}{\partial x_l} = \frac{\partial {}^t \mathbf{z}_e}{\partial x_l} \mathbf{P} + {}^t \mathbf{k}_p^* \frac{\partial \langle \mathbf{KP} \rangle}{\partial x_l} \mathbf{P} \quad (4.48)$$

4.3.4 Structural Theory and TFA

Thermoeconomic Functional Analysis (TFA) establishes that each unit of the system has a function or purpose: in other words, a product. For example, the function of the turbine in the case of the gas turbine analyzed in our example is the work obtained when the gases expand through it. The system is depicted using a scheme called a functional diagram. This diagram represents the units, and the relations among them, and between them and the environment. Each unit uses a set of functions or resources from the environment or from other units to obtain its function or product. These functions are totally or partially consumed in other units and are used again to produce new functions. The functions of several units are sometimes joined in a fictitious unit called a junction, and sometimes a function is distributed to several units using another fictitious device called a branching. To do this, it is necessary to analyze the system carefully, and decompose it into several units according to the functions that are obtained.

The productive structure, as for example the one proposed for the analyzed cogeneration plant, is a fundamental tool of the structural theory, and is based on the functional diagram that was earlier proposed as an analysis tool for the thermoeconomic as well as the functional analysis.

In the next paragraphs, it is proved that the costs obtained by thermoeconomic functional analysis and those obtained by the structural theory are equivalent.

Thermoeconomic functional analysis is oriented towards optimization of complex systems and applies the Lagrange method. The objective function to be minimized is the total cost of the system. The constraint(s) are classified into two groups:

- For each unit the input resources are expressed as a mathematical function of the outputs (or products) and a set of internal variables. Using the nomenclature of the fuel-product model, Eq. (36a) expresses this constraint with the mathematical formalism of the structural theory. Note that in general linearity is not required by either the general formulation of the TFA, or the general formulation of the structural theory.
- The second type of constraint concerns the connections between the units: how a product or an outlet flow of a system is identified as an input or resource of another unit. In the context of the fuel-product model these restrictions are expressed by Eq. (36b).

As was proved in section 3.2, the marginal costs defined in the structural theory coincide with the multipliers of the previous Lagrange function, which is in turn similar to the Lagrange function defined in thermoeconomic functional analysis. Thus, the marginal costs calculated by the structural theory are the same as those calculated by thermoeconomic functional analysis.

4.4 Closure

The benefit of the unification of different methodologies in a common mathematical formalism is evident, because it provides a common basis of comparison: that is, it allows one to compare them using the same concepts and nomenclature.

Thus, when different thermoeconomic methodologies are compared using the mathematical formalism of the structural theory as a common thermoeconomic language, it is shown that the differences between all of them are basically:

- The aggregation level employed in the analysis; and
- The way of connecting the components in the productive structure: that is, the way the plant resources are distributed among the plant components, which is dependent on the fuel–product definitions and the auxiliary equations needed to calculate the costs.

An important characteristic of the structural theory as is that it is a rigorous procedure. When costs are calculated in this way it is obvious that they are a direct consequence of the thermoeconomic model, that is, a direct consequence of the characteristic equations describing the behavior of the components. The better these equations represent the behavior of the plant units, the better the cost information that will be obtained. In other words, the more physical and realistic information is contained in the characteristic equations, the more physical significance the calculated costs will contain.

With the structural theory, we have available all the tools needed for performing a complete thermoeconomic analysis of a complex system, from the point of view of either cost accounting methods or certain optimization methodologies. There are different applications of the results obtained applying the structural theory:

- Rational price assessment of plant products based on physical criteria,
- Energy audits.
- Evaluation of various design alternatives or operational decisions and profitability maximization.
- Optimization of specific process unit variables to minimize final product costs and save resource energy, i.e. global and local optimization.
- Detection of inefficiencies and calculation of their economic effects in operating plants: in other words, plant operation thermoeconomic diagnosis.

Thermoeconomic Diagnosis

This chapter introduces a methodology, based on the Structural Theory of Thermoeconomics and Symbolic Thermoeconomics, to the operation diagnosis of energy systems. The word *diagnosis* applied to energy systems means the art of discovering anomalies by monitoring the operating conditions through on-board measures.

Diagnosis is always performed through comparison of at least two states: the actual state of the plant, which is to be examined, and a reference state, which is a condition without any anomalies. The presence of anomalies in the actual state determines deviations in some of the measured quantities with respect to the reference values.

The aim of techniques adopted in power plants, usually consists of predicting failures in order to avoid them. A more accurate analysis could help the plant management to understand more about reductions in the process efficiency, allowing to locate the components responsible for anomalous behaviour. All these techniques are deductive, since they require the knowledge of the effect of possible anomalies on the thermodynamic quantities describing the plant behaviour.

Thermoeconomic diagnosis belongs to this type of methods, but its objective is most general, consisting in the detection of the efficiency deviation, the location of the main causes and the quantification of its effects in terms of additional fuel consumption or economic impact.

5.1 Thermoeconomic Diagnosis

According to the ASME guide PTC-PM, a diagnosis procedure must have a deductive nature based on the observed symptoms. It must be flexible enough, and must recommend new tests to isolate causes and inform whether these tests are cost-effective or not. The methodology should have the following steps:

- Identification of components and degradation symptoms.
- Clear description of the symptoms to allow a simple problem recognition.
- Evaluation of the deterioration mechanisms and the root causes.
- Validation and conclusions.

As a result, a diagnostic procedure should yield those specific recommendations to change operating strategies, maintenance actions and components replacement.

The objective of a monitoring system is the efficiency improvement or in other words detection of efficiency deviations. A 3% deviation of efficiency with respect to a reference conditions, is quite easily detected by the operator, therefore a monitoring system must detect losses in the range of 0.25–0.5%. Values under this range become difficult to identify because of the instrumentation accuracy. Moreover it is very difficult to locate and to find the real causes of all effects that can simultaneously occur. This is due to the high complexity of interrelations among components in a power cycle. A successful interpretative procedure will reduce the non-accountable losses to a minimum, and will put forward the ultimate causes of component degradation. According with these ideas a diagnosis methodology will require:

Data Acquisition System: to monitoring the power plant, including data filtering, consistence checking and historical storage.

Performance Tests: a procedure, normally based on performance test codes, that determines the actual state of the plant with the higher attainable accuracy, with regard to the available instrumentation.

State of Reference: a validated model of the plant which represents the state of reference for any operation mode, environmental conditions or feedstock compositions.

Thermoeconomic Diagnosis Model: that allocates and assesses the increase of resources consumption compared to the one foreseen by the state of reference and explains the underlying causes.

This paper focuses on the last point, the description of theoretical basis and the practical procedures of a thermoeconomic methodology for operation diagnosis.

In order to clarify the concepts introduced in the paper, we will use the example of a cogeneration gas turbine described in the previous articles. The control parameter values and fuel – product values for reference and operation conditions, used in the example – are shown in Tables 1 and 2.

Table 5.1. Model Operation Variables

Description	x^0	x
Combustor pressure losses (ΔP_1)	0.05	0.052
Compressor isentropic efficiency (η_{cp})	0.85	0.84
Turbine isentropic efficiency (η_{tg})	0.87	0.86
Inlet turbine gases temperature (T_3)	850 °C	855 °C
Compression ratio (P_2/P_0)	5	5
Combustor 1 st law efficiency (η_{cb})	0.98	0.975

Table 5.2. Design and operation values.

	Design			Operation		
	P (kW)	I (kW)	k	P (kW)	I (kW)	k
Combustor	6630.9	5150.1	1.7767	6815.2	5472.4	1.8030
Compressor	2594.8	382.2	2.3464	2662.7	449.0	2.4641
Turbine	5477	306	2.0451	5611.8	338.1	2.1085
HRSG	2355	1087.7	2.8315	2355	1173.0	2.9793

5.1.1 Technical Exergy Saving and Fuel Impact

Thermoeconomic diagnosis is a Second Law based technique oriented to operation analysis. The exergy balance of an installation allows us to allocate and calculate irreversibilities in the production process and to identify the equipment which affect overall efficiency and the reasons thereof. This information, although useful, has proved not to be enough. In practice, when attempting to achieve energy savings in an installation, we must consider that not all irreversibility can be avoided. The potential exergy saving is limited by technical and economic constraints. Thus, the technical possibilities for exergy savings, which is called technical exergy saving, are always lower than the theoretical limit of thermodynamic exergy losses. From this perspective, in the plant of our example, we can see in Table 2 that only 506 kW, out of the 7.432 MW of the total irreversibilities of the plant, can be saved with respect to reference conditions.

Therefore, the additional fuel consumption can be expressed as the difference between the resources consumption of the plant in operation and the resources consumption for a reference or design condition, with the same production objectives, i.e. with the same total plant product.

$$\Delta F_T = F_T(x) - F_T(x_0) \quad (5.1)$$

It can be broken up into the sum of the irreversibilities of each component:

$$\Delta F_T = \Delta I_T = \sum_{j=1}^n (I_j(x) - I_j(x_0)) = \sum_{j=1}^n \Delta I_j \quad (5.2)$$

However, the local exergy savings which can be achieved in the different units or processes of an installation are not equivalent. The same decrease in the local

irreversibility of two different components leads, in general, to different variations of the total plant energy consumption. It is shown in the *fuel impact formula*, presented in the article *Symbolic Thermoeconomic Analysis of Energy Systems*, that expresses the increase of resources consumption in the plant, as a function of the marginal exergy consumption of each individual component of the plant:

$$\Delta F_T = \sum_{i=1}^n \left(\sum_{j=0}^n k_{p,j}^*(x) \Delta \kappa_{ji} \right) P_i(x_0) + k_{p,i}^* \Delta P_{s,i} \quad (5.3)$$

The variation of the marginal exergy consumption of each component increases its resources consumption and then its irreversibilities in a quantity $\Delta \kappa_{ji} P_i^0$, which is called, *malfunction*. Consequently, it implies an additional consumption of the external resources given by $k_{p,j}^* \Delta \kappa_{ji} P_i^0$, which is called the *malfunction cost*. Therefore, the total fuel impact can be written as the sum of the fuel impact or malfunction cost of each component, as shown in Eq. (4.3).

In order to analyze the impact on resource consumption of a plant, we need to know the design and operation values of the irreversibilities, product, unit exergy cost for design and operation, and the increase of the marginal exergy consumption of each component of the plant. A performance test or a simulator model together with the fuel-product model of the plant defined in figure 1, provide the values shown in Table 3.

$$\Delta \langle \mathbf{KP} \rangle = \begin{bmatrix} 0.02629 & 0 & 0 & 0 \\ 0 & 0 & 0.00348 & 0.02652 \\ 0 & 0 & 0.00089 & 0.00972 \\ 0 & 0.02135 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1 compares in a bar graph the malfunction cost and the irreversibility increase or technical saving of each component. It shows that the irreversibility increase and the malfunction are mainly located in combustor, meanwhile malfunction cost appears in all components.

5.2 Malfunction and Disfunction Analysis

In this section we will analyze the causes of the irreversibilities increase, and the relation with the malfunction costs. We have shown that there is no direct relationship between the increase of the irreversibilities and its fuel impact. The more advanced the production process is, the greater is the cost of the irreversibility malfunction and as a consequence the greater its fuel impact is.

Furthermore, the degradation of a component forces the other components to adapt their behaviour in order to maintain their production conditions, and as a consequence their irreversibilities are modified. Figure 4.2, shows how an increase of the unit consumption of a component increases, not only the irreversibilities of such component, but also the irreversibilities of the previous component.

The irreversibility increase of a generic system's component is given by:

$$\Delta \mathbf{I} = \Delta \mathbf{K}_D \mathbf{P}^0 + (\mathbf{K}_D - \mathbf{U}_D) \Delta \mathbf{P} \quad (5.4)$$

From the above expression, we can distinguish two types of irreversibilities:

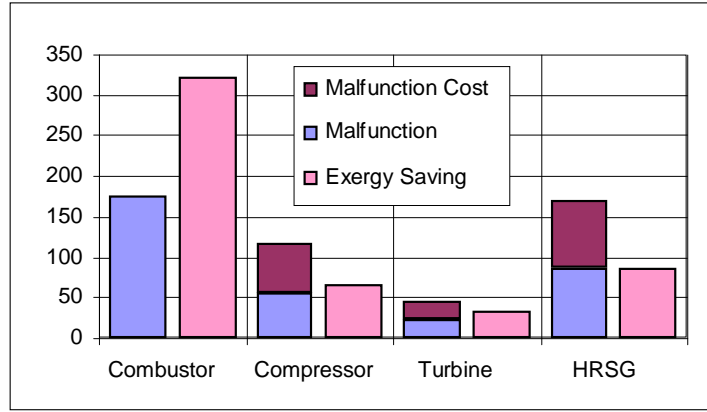


Figure 5.1. Malfunction cost and exergy saving.

- *Endogenous irreversibility* or *malfunction* produced by an increase of the unit consumption of the component itself:

$$MF_{ji} = \Delta\kappa_{ji}P_i^0 \quad MF_i = \sum_{j=0}^n MF_{ji} \quad (5.5)$$

- *Exogenous irreversibility* or *dysfunction* induced in the component by the malfunction of other subsystems, which forces it to consume more local resources in order to obtain the additional production required by the other components:

$$DF_i = (k_i - 1)\Delta P_i \quad (5.6)$$

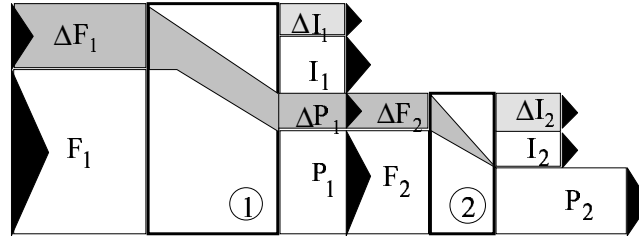


Figure 5.2. Irreversibility coefficients for operation state

The *malfunction* only affects the behavior of the components; the *dysfunction* arises in how the components adapt themselves to maintain the total production.

Now we will study the causes and effects of the system irreversibilities and introduce a new method to compute the fuel impact of a *malfunction* and its effect or, in other words, to compute also the *dysfunction* on the rest of the system components.

The irreversibility increase Eq. (4.4) can be written in terms of the unit consumption increase, by means of:

$$\Delta I = (\Delta K_D + |I|) \Delta \langle KP \rangle P^0$$

or in scalar format:

$$\Delta I_i = \sum_{j=0}^n \Delta\kappa_{ji}P_i(x_0) + \sum_{j,h=1}^n \phi_{ih}(x) \Delta\kappa_{hj}P_j(x_0) \quad i = 1, \dots, n \quad (5.7)$$

The first part of the previous expression corresponds to the component *malfunction*, and the last part to the *dysfunction*. If we denote:

$$DF_{ij} = \sum_{h=1}^n \phi_{ih}(x) \Delta \kappa_{hj} P_j(x_0) \quad (5.8)$$

$$DF_i = \sum_{j=1}^n DF_{ij}$$

DF_{ij} represents irreversibility increase of the i -th component, due to the malfunctions of the j -th component. The ϕ_{ih} are the coefficients of the irreversibility matrix operator $[\mathbf{I}]$ for the actual operation values, shown in Table 4. The above expression shows how a malfunction $P_j \Delta \kappa_{hj}$, on the j -th component, generates a dysfunction on the i -th component proportional to the ϕ_{ih} coefficients, which represent the weight of the malfunction effect. These coefficients do not depend on the malfunction amount, but only on the unit consumption of the components in an operating state. Therefore, the dysfunction can not be corrected by itself, only decreasing the malfunction, which generated it.

The technical exergy saving of component i -th, Eq. (4.7) can be written as the sum of its malfunction and the dysfunction generated by other components of the system:

$$\Delta I_i = MF_i + \sum_{j=1}^n DF_{ij} \quad i = 1, \dots, n \quad (5.9)$$

and the fuel impact in agreement with Eq. (4.2) can be written in terms of malfunction and dysfunction as follows:

$$\Delta F_T = \sum_{i=1}^n \Delta I_i = \sum_{i=1}^n \left(MF_i + \sum_{j=1}^n DF_{ij} \right) \quad (5.10)$$

Figure 4.3 shows a graph describing the causes of the increase of irreversibility, as sum of the malfunctions and the dysfunctions generated by the rest of the components, which mainly apply to the combustor.

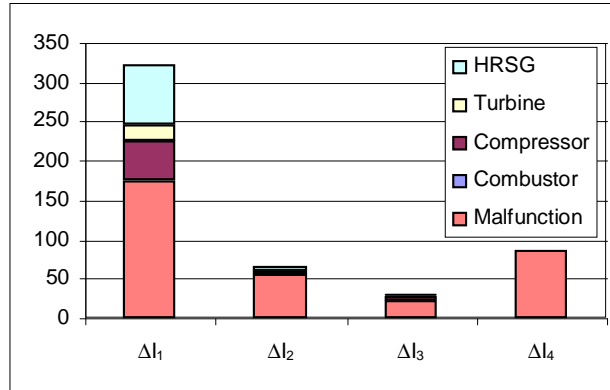


Figure 5.3. Irreversibility increase analysis

We have defined above the *malfunction cost*, as the malfunction multiplied by the

unit cost of the additional resources required:

$$MF_{ji}^* = k_{p,j}^*(x) \Delta \kappa_{ji} P_i(x_0) / MF_i^* = \sum_{j=0}^n MF_{ji}^* \quad (5.11)$$

and the malfunction cost of a component as the sum of the component malfunction cost. The total fuel impact is written as:

$$\Delta F_T = \sum_{i=1}^n MF_i^* \quad (5.12)$$

The unit exergy cost of a product can be written in terms of the irreversibility coefficients, by means of the expression, introduced in a previous chapter:

$$k_{p,j}^* = 1 + \sum_{i=1}^n \phi_{ij} \quad (5.13)$$

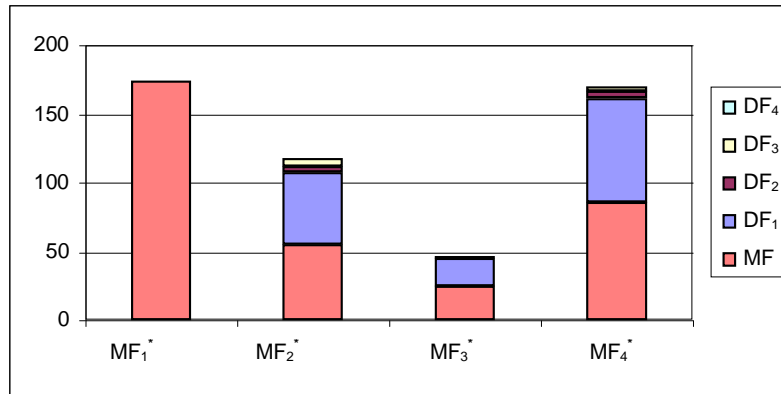
Therefore Eq. (4.11) can be rewritten as:

$$MF_i^* = MF_i + \sum_{h=1}^n DF_{hi} \quad (5.14)$$

It means that the malfunction cost of each component is given by the sum of the malfunction and the dysfunction generated by this component. Figure 4.4 shows a graph of the malfunction cost of each component as sum of the malfunction and the dysfunction or irreversibility generated in other components, as complementary view of Figure 4.3.

The dysfunction generated by a component, just as the fuel impact, depends on the malfunction and the position of the component in the productive process, characterized by the unit cost of the resources required by the component.

Figure 5.4. Malfunction cost analysis



The relation between irreversibility increase and fuel impact can be represented in a double input table, as shown in Table 4.5, that includes only the malfunctions and dysfunctions of the components. The columns of the table represent the malfunction costs and the rows the irreversibility increase.

Table 4.5 together with Figures 4.3 and 4.4, show that 63% of the irreversibility increase is located in the combustor, but malfunction in the combustor represents

Table 5.3. Malfunction or disfunction table

	Combustor	Compressor	Turbine	HRSG	DF	MF	Total
ΔI_1	0.000	52.014	20.684	75.253	147.951	174.336	322.287
ΔI_2	0.000	4.268	1.267	5.919	11.454	55.389	66.843
ΔI_3	0.000	5.119	0.529	2.471	8.119	23.964	32.083
ΔI_4	0.000	0.000	0.000	0.000	0.000	85.329	85.329
DI	0.000	61.400	22.480	83.643	167.524		
MF	174.336	55.389	23.964	85.329		339.018	
Total	174.336	116.790	46.444	168.972			506.543

only 34% of the total fuel impact, the rest of its irreversibility increase is due to structural malfunctions or dysfunctions generated by the malfunctions, 37% of the total fuel impact, of the rest of components. According to energy saving analysis the problem would be located in the combustor, but malfunction cost analysis indicates that there will be other locations also in other components. But, where is the origin of the fuel impact?

5.3 Malfunction Causes Analysis

The proposed thermoeconomic methodology, is a powerful tool to find the causes of the irreversibilities and quantify them in terms of fuel impact. The malfunction/dysfunction analysis is an upstream analysis of the productive process. It analyzes how the previous components on the plant are affected by a component malfunction. But this model is not enough to determine the origin of malfunctions.

The problem to determine the malfunction causes, is that we cannot operate a unit of the plant using the unit exergy consumption, they are not real control parameters of the plant, but function of the real control variables as pressure ratios, temperatures, mass flow rates.

Thus, is it possible to express the fuel impact as the sum of the contributions of the control variables? To do that, we need to relate the unit exergy consumption increase or the malfunctions as a sum of the contributions of each control variable. As first order approximation, we can write:

$$\Delta \kappa_{ij} \approx \sum_{l=1}^r \frac{\partial \kappa_{ij}}{\partial x_l} \Delta x_l$$

$$MF_{ij} \approx \sum_{l=1}^r \frac{\partial \kappa_{ij}}{\partial x_l} P_j^0 \Delta x_l$$
(5.15)

By using a reference state model simulator, it is possible to fit the malfunctions, with a polynomial on the form:

$$MF_{ij}^l \cong \sum_p \alpha_{ij,p}^l (x_l - x_l^0)^p$$
(5.16)

Therefore the malfunction can be evaluated by using the expression:

$$MF_{ij} = \sum_{l=1}^r MF_{ij}^l + \varepsilon_{ij} \quad (5.17)$$

where ε_{ij} is the fitting error. This error could be interpreted as non-identified malfunction, due to interaction between control parameters. Table 6 shows the malfunction table, which contains the malfunction associated to each local variable, and the error obtained, using a polynomial fitting of the first degree.

The above table shows, that the decrease of the gas turbine efficiency is the main cause of the malfunctions, instead of the parameters associated to the combustor as pressure losses, or efficiency.

Once the relationship between malfunction and the operating parameters is known, the method proposed in previous sections could be applied to distinguish the effect of a control parameter on the internal economy of the component. The malfunction cost of each component could be obtained as:

$$MF_i^* \cong \sum_{l=1}^r \sum_{j=0}^n k_{p,j}^* MF_{ji}^l \quad (5.18)$$

and the irreversibility increase, or technical exergy saving, is obtained by means of:

$$\Delta I_i \cong \sum_{l=1}^r \left(MF_i^l + \sum_{j,h=1}^n \phi_{ih} MF_{hj}^l \right) \quad (5.19)$$

Figure 4.5 shows the causes of the irreversibility increases of each component, and it has been explained before the main cause of irreversibility increase in the combustor is the decrease of the gas turbine efficiency, which can provoke an increase in the mass flow rate to maintain the production.

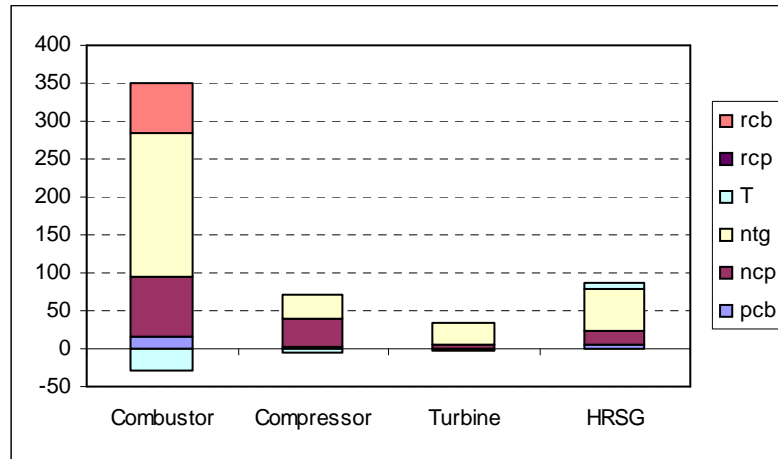


Figure 5.5. Irreversibility increase causes

The value of the non-identified malfunctions can be reduced. Consider that the contribution of each control variable is not additive, but due to the interaction between variables it is weighted by an interaction factor β_l which could be determined by a least squares procedure:

$$\min_{\beta_l} \sum_{i,j} \left(MF_{ij} - \sum_l \beta_l MF_{ij}^l \right)^2 \quad (5.20)$$

Figure 6 resumes the diagnosis procedure described in the paper, which combines simulation techniques and thermoeconomic analysis. It shows the fuel impact due to each operational parameter, and the malfunction cost associated to each component.

The degradation or improvement of a component causes a variation in the exergy consumption of the component itself, that we call *intrinsic malfunction*. Table 6, shows in bold characters the intrinsic malfunctions of each component. An intrinsic malfunction not only has an impact upstream in the process, that we have called *dysfunction* or *structural malfunction*, but also downstream. The variation of the input conditions in a component will affect its efficiency, to a greater or lesser extent which in turn, will affect the next component. Not only are there dysfunctions when there is an intrinsic malfunction; there are also *induced malfunctions* that can decisively affect the behavior of a plant. In our example, 1% of variation in the isentropic efficiency of the turbine causes an irreversibility increase of 25 kW in the gas turbine, but this downstream effect on the total plant is 12 times greater. Only 25% of the additional resources consumption is due to the intrinsic malfunctions, 42% is due to induced malfunctions and 33% to structural malfunctions.

The type of malfunction is related to the behavior of the operating parameters regarding the effect on the efficiency of the component of the system. If the parameters are *local*, it means that malfunctions are produced only on the component to which the parameter is related. For example the efficiency or the pressure losses on the combustor, shown in Table 4.6, the malfunctions are intrinsic, and thermoeconomic analysis based on malfunction cost could affect its fuel impact. But if the parameters are *global*, they cannot be associated to a specific component, and generate malfunctions in several components of the plant. In general the malfunctions are of both types: intrinsic and induced. Conventional thermoeconomic analysis cannot assess with accuracy its fuel impact. Therefore thermoeconomic diagnosis depends on the choice of the operation parameters and the aggregation level of the productive structure. A suitable choice of the model that makes *local* most of the operational parameters will enhance the thermoeconomic diagnosis procedures.

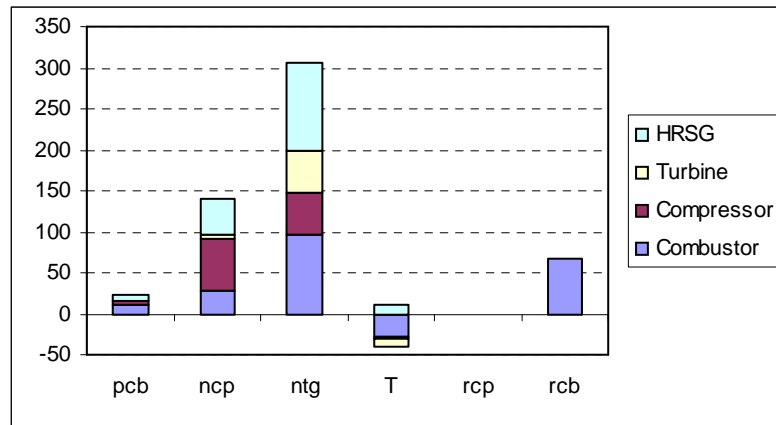


Figure 5.6. Malfunction cost of operational parameters

5.4 Closure

This paper has presented the application of thermoeconomic diagnosis to enhance energetic resources utilization and thermal power plant management.

The fundamental feature consists of a detailed analysis of the equipment interactions and its productive structure. Once reliable input data have been provided, malfunction quantification and the causes can be determined by comparison of the steady state obtained from a performance test and a reference state given by a model simulator.

The thermoeconomic diagnosis methodology explained in this article is based on the Structural Theory of Thermoeconomics and Symbolic Thermoeconomics. It is possible to predict exactly the total additional fuel consumed when a plant malfunctions or inefficiencies occur. The most important fact is not to predict the extra fuel consumed, which can be also calculated by a simulator model of the plant, but to explain component by component the additional fuel consumption and to assess for each one its corresponding responsibility or malfunction cost. This information cannot be obtained with a conventional simulator, but by applying thermoeconomic analysis.

Diagnosing an energy system consists of comparing an actual state with a reference one and locating the final causes of its deviation. This implies it is necessary to identify how, where and how much of the additional consumed resources could be saved. A conventional exergy analysis comparing the irreversibilities at the components level is not at all adequate. This is because many irreversibilities occurring in a system's components are not due to an actual malfunction of that component but to the influence of other components.

A first variational analysis is then required by considering that during a malfunction the other components only adapt their resources and their production to the new situation without changing their efficiency. The key of this analysis is to calculate the dysfunction DF_{ij} i.e. the irreversibility increase on the i -th component generated by a malfunction on the j -th component. Therefore the irreversibility increase of a component will be its own malfunction plus all the dysfunction generated by the rest of the components, and the cost of malfunction is the malfunction plus the dysfunctions they generate in the rest of the components.

A second approach considers that the degradation in a component, *intrinsic malfunction*, can modify the efficiency of the other plant components, *induced malfunction*. The way to solve is to build a *malfunction matrix*, by the help of a model simulator or an additional external knowledge not provided by classic thermoeconomic analysis. Then it is possible to identify the operating parameters that potentially causes deviation, and calculate the intrinsic and induced malfunction, and then their fuel impact.

An operation diagnosis methodology requires both a thermoeconomic analysis and a model simulator, in order to identify and quantify the origin of the irreversibility increase at the components level, and then to take the actions required to reduce the resources consumption energy.

However, the thermoeconomic diagnosis technique presented in this article it is not completed. Diagnosis is the art of discovering and understanding the real causes of malfunction and quantifying their effects. This is a very complex task because many different factors occur simultaneously. When the behavior of a component is modified it can be provoked by several different situations that not necessarily correspond with an intrinsic malfunction of the referred component. For instance, partial load operation, ambient conditions, the fuel plant quality modifications and the control system of the plant can provoke deviations of the behavior and the efficiency of a component. Applying the technique presented in this paper a fuel impact would be detected due to the efficiency deviation of the referred component, but its malfunction is not intrinsic, i.e. this malfunction has been provoked by the

control system or the different ambient conditions, or the different fuel quality or the partial load. In other words, the problem of identifying and quantifying the intrinsic malfunctions is not entirely solved yet. This is the cornerstone of diagnosis. We now have understanding of the interactions between the different plant components and how a malfunction affects the behavior of the different plant components, but we have not entirely solved yet the problem of identifying the real causes provoking the malfunctions. The Ph.D. Thesis developed by Verda has solved this problem in the case of the control system of the plant. But further research work is required.

Exergy Analysis Fundamentals

This appendix briefly surveys exergy principles and illustrates their use. To explore the subject in greater depth, readers should refer to [3, 9] or [13],

A.1 Engineering thermodynamics principles

The exergy concept is developed using extensive balances of mass, energy and entropy, together with property relations. Here, we will consider properties of simple compressible substances exclusively. Engineering applications involving exergy are generally analyzed on a control volume basis, see figure A.1. Accordingly, the control volume formulation of the mass, energy and entropy balances presented in this section play important roles. These are provided here in the form of overall balances assuming one-dimensional flow.

Mass rate balance

For applications in which inward and outward flows occur, each through one or more ports, the extensive property balance mass expression the conservation mass principle takes the form

$$\frac{dm}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \quad (\text{A.1})$$

where dm/dt represents the time rate of change of mass contained within the control volume, \dot{m}_i denotes the mass flow rate at an inlet port, and \dot{m}_e denotes the mass flow rate at an exit port.

Energy rate balance

Energy is a fundamental concept of thermodynamics and one of the most significant aspects of engineering analysis. Energy can be stored within systems in various macroscopic forms: kinetic energy, gravitational potential energy, and internal energy. Energy can also be transformed from one form to another and transferred between systems. Energy can be transferred by work, by heat transfer, and flowing matter. The total amount of energy is conserved in all transformations and transfers.

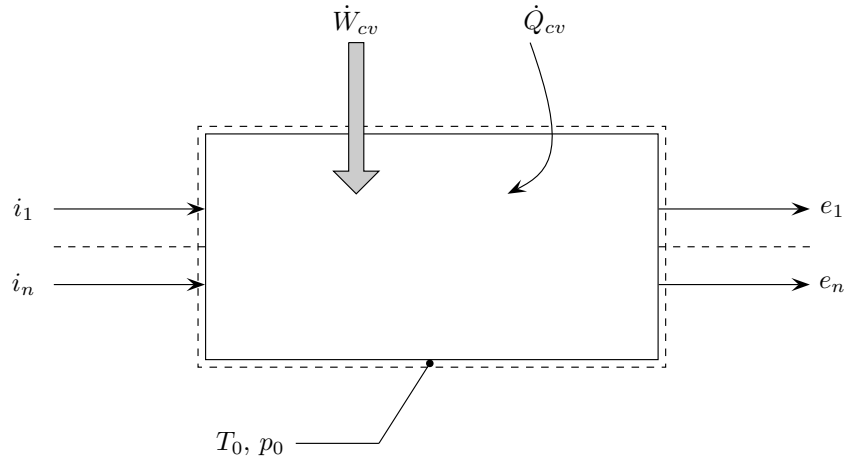


Figure A.1. Control volume at steady state

On a time-rate basis, the extensive property balance for the energy of a closed system takes the form

$$\frac{d(U + KE + PE)}{dt} = \dot{Q} - \dot{W}, \quad (\text{A.2})$$

where U , KE , and PE denote, respectively, the internal energy, kinetic energy, and gravitational potential energy. The terms \dot{Q} and \dot{W} account, respectively, for the net rates of energy transfer by heat and work. In equation A.1, energy transfer by heat to the system is considered positive and energy transfer by work from the system is considered positive.

Energy can enter and exit control volumes by work, by heat transfer and by flowing matter. Because work is always done on or by a control volume where matter flows across the boundary, it is convenient to separate the work rate into two contributions. One contribution is the work rate associated with the force of the fluid pressure as mass is introduced at the inlet and removed at the exit. This is commonly referred to as flow work. The other contribution denoted as \dot{W}_{cv} , includes all other work effects, such as those associated with rotating shafts, displacement of the boundary, and electrical effects. On a one-dimensional flow basis, the total work rate associated with a control volume is

$$\dot{W} = \dot{W}_{cv} + \sum_e \dot{m}_e p_e v_e - \sum_i \dot{m}_i p_i v_i \quad (\text{A.3})$$

where p_i and p_e denote the pressures and v_i and v_e denote specific volumes at the inlets and exits, respectively. Energy also enters and exits control volumes with flowing streams of matter. The specific energy (energy per unit mass) is the sum of the specific internal energy u , the specific kinetic energy $V^2/2$, and the specific gravitational potential energy gz . Thus, on a one-dimensional flow basis, the rate at which energy enters with matter at inlet i is $\dot{m}_i (u_i + V_i^2/2 + gz_i)$.

Collecting results, the following form of the control volume energy rate balance evolves:

$$\begin{aligned} \frac{d(U + KE + PE)}{dt} = & \dot{Q} - \dot{W}_{cv} + \sum_i \dot{m}_i (u_i + p_i v_i + V_i^2/2 + gz_i) \\ & - \sum_e \dot{m}_e (u_e + p_e v_e + V_e^2/2 + gz_e) \end{aligned} \quad (\text{A.4})$$

Introducing the specific enthalpy h ($h = u + p v$), the energy rate balance becomes

$$\begin{aligned} \frac{d(U + KE + PE)}{dt} = \dot{Q} - \dot{W}_{vc} + \sum_i \dot{m}_i (h_i + V_i^2/2 + gz_i) \\ - \sum_e \dot{m}_e (h_e + V_e^2/2 + gz_e) \end{aligned} \quad (\text{A.5})$$

Entropy rate balance

An extensive property balance also can be written for entropy. The closed system entropy balance reads

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + S_g \quad (\text{A.6})$$

Because entropy is a property, the entropy change on the left side of equation (A.6) can be evaluated independently of the details of the process, knowing only the end states. However, the two terms on the right side depends explicitly on the nature of the process and cannot be determined solely from knowledge of the end states.

The first term on the right side of equation (A.6) is associated with heat transfer to or from the system during the process. This term can be interpreted as the entropy transfer accompanying heat transfer. The direction of entropy transfer is the same direction of heat transfer, and the same sign convention applies as for heat transfer. The second term as the right side, S_g , accounts for entropy generated within the system owing to irreversibilities. The term S_g is positive, the Second Law of thermodynamics can be interpreted as specifying that entropy is generated by irreversibilities and conserved only if in the limit as irreversibilities are reduced to zero. Because S_g measures the effect of irreversibilities present within a system during a process, this value depends on the nature of the process and not solely on the end states, and thus is not a property. When irreversibilities are absent during a process, no entropy is generated within the system and the process is said to be reversible.

A rate form of the closed system entropy balance that is frequently convenient is

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_g \quad (\text{A.7})$$

where dS/dt is the time rate of change of entropy of the system. The term \dot{Q}_j/T_j represent the time rate of entropy transfer through the portion of the boundary whose instantaneous temperature is T_j . The term \dot{S}_g accounts the time rate of entropy generation due to irreversibilities within the system.

As for the case of energy, entropy can be transferred into or out a control volume by streams of matter, This is the principal difference between the closed system and control volume forms. Accordingly, for control volumes the counterpart of equation (A.7) is

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{S}_g \quad (\text{A.8})$$

where dS/dt represents the time rate of change of entropy within the control volume. The terms $\dot{m}_i s_i$ and $\dot{m}_e s_e$ account, respectively, for rates of entropy transfer into and out the control volume accompanying mass flow. \dot{Q}_j represents the time rate of heat transfer at the location on the boundary where instantaneous temperature

is T_j . \dot{S}_g denotes the time rate of entropy generation due to irreversibilities within the control volume.

When applying the entropy balance, in any of its forms, the objective is often to evaluate the entropy generation term. However, the value of the entropy generation for a given process of a system usually does not have too much significance by itself. The significance is normally determined through comparison: The entropy generation within a given component might be compared with the entropy generation values of the other components included in a system formed by these components. By comparing entropy generation values, the components where appreciable irreversibilities occur can be identified and rank-ordered, allowing attention to be focused on the components that contribute most heavily to inefficient operation of the overall system.

To evaluate the entropy transfer term of the balance requires information about both the heat transfer and the temperature on the boundary where the heat transfer occurs. The entropy transfer term is not always subject to direct evaluation, however, because the required information is either unknown or not defined, such as when the system passes through states sufficiently far from equilibrium. In practical applications, it is often convenient, therefore, to enlarge to include enough of the immediate surroundings that the temperature on the boundary of the enlarged system corresponds to the ambient temperature, T_0 . The entropy transfer rate is then simply \dot{Q}/T_0 . However, as the irreversibilities present would not be just those for the system of interest but those for the enlarged system, the entropy generation term would account for the effects of internal irreversibilities within the system and external irreversibilities with that portion of the surroundings included within the enlarged system.

Steady-state rate balances

For control volumes at steady state, the identity of the matter within the control volume changes continuously, but the total amount of mass remains constant. At steady state, equation (A.1) reduces to:

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e \quad (\text{A.9})$$

As steady state, the energy rate balances given by (A.5) reduces to read

$$\dot{W}_{vc} - \dot{Q} = \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) \quad (\text{A.10})$$

and the equation (A.8) is written as

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{S}_g \quad (\text{A.11})$$

When supplemented by appropriate property relations, the previous equations allow control volumes at steady state to be analyzed.

A.2 Defining Exergy

An opportunity for doing work exists whenever two systems at different states are placed in communication, for in principle, work can be developed as two

are allowed to come into equilibrium. When one of the systems is a suitably idealized system called an environment and the other is some system of interest, *exergy* is the maximum theoretical useful work obtainable as the systems interact to equilibrium, heat transfer occurring with the environment only. Alternatively, exergy is the minimum theoretical useful work required to form a quantity of matter from substances present in the environment and bring the matter to specified state. As exergy is a measure of the departure of the state of the system from that of environment, exergy is an attribute of the system and environment together. Once the environment is specified, however, a value can be assigned to exergy in terms of property values for the system only, so exergy can be regarded as an extensive property of the system.

Exergy can be destroyed and generally is not conserved. A limiting case is when exergy would be completely destroyed, as occurs when a system comes into equilibrium with the environment spontaneously with no provision to obtain work. The capability to develop work existing initially would be completely wasted in the spontaneous process. Moreover, because no work need be done to effect such a spontaneous change, it may be concluded that the value of exergy is at least zero and therefore cannot be negative.

Environment and Dead state

Any system, whether a component in larger system such as a steam turbine in a power plant or the power plant itself, operates within surroundings of some kind. Distinguishing between the environment and the system's surroundings is important. The term *surroundings* refers to everything not included in the system. The term *environment* applies to some portion of the surroundings, that the intensive properties of each phase of which are uniform and do not change significantly as a result of any process under consideration. The environment is regarded as free of irreversibilities. All significant irreversibilities are located within the system and its immediate surroundings.

Because the physical world is complicated, models with various levels of specificity have been proposed for describing the environment. In the present discussion the environment is modeled as a simple compressible system, large in extent, and uniform in temperature, T_0 , and pressure, p_0 . In keeping with the idea that the environment has to do with the actual physical world, the values for p_0 and T_0 used throughout a particular analysis are normally taken as typical environmental conditions, such as one atmosphere and 25°C.

The environment is regarded as composed of common substances existing in abundance within the atmosphere, the oceans, and the crust of the Earth. The substances are in their stable forms as they exist naturally, and developing work from interactions – physical or chemical – between parts of the environment is not possible. Although its intensive properties do not change, the environment can experience changes in its extensive properties as a result of interactions with other systems. Kinetic and potential energies are evaluated relative to coordinates in the environment all parts of which are considered to be at rest with respect to one another. Accordingly, a change in the energy of the environment can be change in its internal energy only.

When the pressure, temperature, chemical composition, velocity, or elevation of a system is different from the environment, an opportunity to develop work exists. As the system changes state toward that of the environment, the opportunity diminishes, ceasing to exist when the system and the environment, at rest relative to one another, are in equilibrium. This state of the system is called the *dead state*.

At the dead state, the conditions of mechanical, thermal and chemical equilibrium between the system and the environment are satisfied: the pressure, temperature, and chemical potentials of the system are equal those the environment, respectively. In addition, the system has zero velocity and zero elevation relative to coordinates in the environment. Under these conditions, no possibility exists of spontaneous change within the system or the environment, nor can they interact.

Another type of equilibrium between the system and the environment can be identifies. This form of equilibrium is restricted, only the conditions of mechanical and thermal equilibrium must be satisfied. This state of the system is called the *restricted dead state*.

Exergy components

In the absence of nuclear, magnetic, electrical, and surface tension effects, the exergy E_{sys} can be divided into four components: physical exergy, E_{sys}^{PH} , kinetic exergy, E^K , potential exergy, E^P , and chemical exergy, E^{CH} , that is

$$E_{sys} = E_{sys}^{PH} + E^K + E^P + E^{CH} \quad (A.12)$$

The sum of the kinetic, potential, and physical exergies is also referred to as thermo-mechanical exergy. Although exergy is extensive property, it is often convenient to work with it on a unit mass basis. The specific exergy, e is given by

$$e_{sys} = a_{sys}^{PH} + \frac{V^2}{2} + g z + E^{CH} \quad (A.13)$$

In equation (A.13), V and z denote velocity and elevation relative to coordinates in the environment. When evaluated relative to the environment, the kinetic and potential energies of the system are, in principle, fully convertible to work as the system is brought to rest relative to the environment, and so they correspond to the kinetic and potential exergies, respectively.

Considering a system at rest relative to the environment ($E^K = E^P = 0$) the physical exergy is the maximum theoretical useful work obtainable as the system passes from the initial state, where the temperature is T and the pressure is p , to the restricted dead state, where the temperature is T_0 and pressure is p_0 . The chemical exergy is the maximum theoretical work obtainable as the system passes from the restricted dead state to the dead state, where it is in complete equilibrium with the environment. The use of the term *chemical* here does not necessarily imply a chemical reaction.

Physical exergy

The physical exergy of a closed system at a specified state is given by the expression

$$E_{sys}^{PH} = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) \quad (A.14)$$

where U , V , and S denote, respectively, the internal energy, volume, and entropy of the system at the specific state, and U_0 , V_0 , and S_0 denote the energy, volume, and entropy of the system when is at the restricted dead state. On mass-unit basis, the physical exergy is

$$e_{sys}^{PH} = (u - u_0) + p_0(v - v_0) + T_0(s - s_0) \quad (A.15)$$

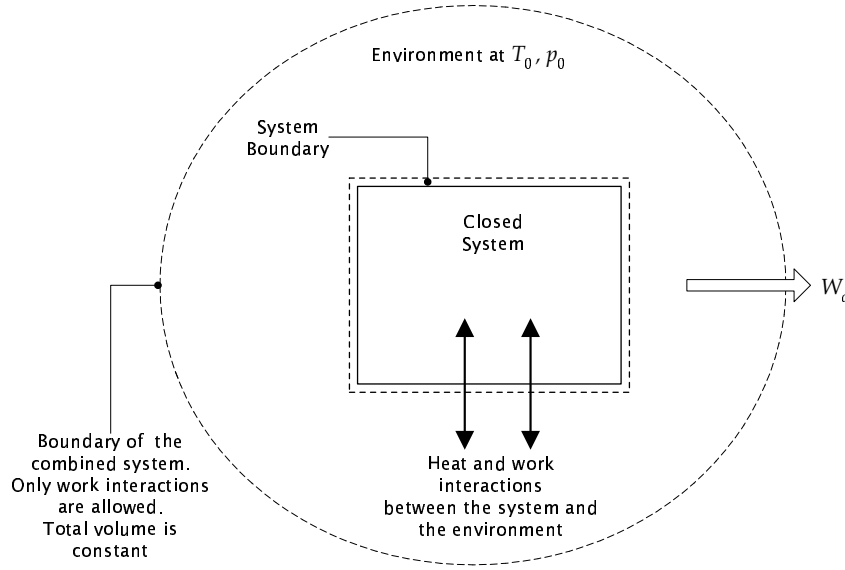


Figure A.2. Combined system of closed system and environment

Equation (A.14) for the physical exergy can be derived by applying energy and entropy balances to the combined system shown in figure A.2, which consists of a closed system and the environment. The system is at rest relative to the environment. As the objective is to evaluate the maximum work that could be developed by the combined system, the boundary of the combined system allows only energy transfers by work across it, ensuring that the work developed is not affected by heat transfer to or from the combined system. And although the volumes of the system and environment may vary, the boundary of the combined system is located so the total volume remains constant. The energy balance for the combined system reduces to

$$\Delta U_c = -W_c$$

where W_c is the work developed by the combined system, and ΔU_c is the internal energy change of the combined system: the sum of the internal energy changes of the closed system and the environment. Accordingly, ΔU_c can be expressed as

$$\Delta U_c = (U_0 - U) + \Delta U^e$$

where ΔU_e denotes the internal energy change of the environment, that could be related to changes in the entropy S^e and volume V^e of the environment through the equation

$$\Delta U^e = T_0 \Delta S^e + p_0 \Delta V^e$$

Collecting the last three equations,

$$W_c = (U - U_0) + p_0(V - V_0) - T_0 \Delta S^e \quad (\text{A.16})$$

This equation gives the work developed by the combined system as the closed system passes to the restricted dead state while interacting only with the environment. Since no heat transfer occurs across its boundary, the entropy balance for the combined system reduces to give

$$\Delta S_c = S_g$$

where S_g accounts for entropy generation within the combined system as the closed system comes into equilibrium with the environment. The entropy change of the

combined system, ΔS_c , is the sum of the entropy changes for the closed system and environment, respectively,

$$\Delta S_c = (S_0 - S) + \Delta S^e$$

Combining the last to equation, and inserting the result into equation (A.16) gives

$$W_c = \underline{(U - U_0) + p_0(V - V_0) - T_0(S - S_0)} - T_0 S_g$$

The value of the underlined term is determined by two states of the closed system – the initial state and the restricted state – and is independent of the process. However, the value of S_g depends on the nature of process as the closed system passes to the restricted dead state. In accordance with the second law this term cannot be negative. Hence the *maximum theoretical value* for the work of the combined system is obtained by setting S_g to zero, leaving

$$W_{c,\max} = (U - U_0) + p_0(V - V_0) - T_0(S - S_0)$$

By definition, the physical exergy, E_{sys}^P , is this maximum value, and equation (A.14) is obtained as the appropriate expression for calculating the physical exergy of a system.

A.2.1 Chemical Exergy

When evaluating chemical exergy – the exergy component associated with the departure of the chemical composition of a system from that of the environment – the substances composing the system must be referred to the properties of a suitably selected set of environment substances. For there to be no possibility of developing work from interactions, physical or chemical, between parts of the environment, it is essential that these environmental reference substances be in equilibrium, mutually and with the rest of the environment. The natural environment is not in chemical equilibrium, however, and a compromise between physical reality and requirements of thermodynamic theory is necessary. Such considerations have led to alternative models for evaluating chemical exergy, and the term *exergy reference environment* is frequently used to distinguish thermodynamic concept from the natural environment. The modeling of exergy reference environments is thoroughly discussed in the literature. For simplicity, the present development features the use of standard chemical exergies determined relative to a standard environment.

Standard chemical exergies are calculated on the basis that the environmental temperature and pressure have standard values and the environment consists of a set of reference substances with standard concentrations reflecting the chemical makeup of the natural environment. The reference substances fall into three groups: gaseous components of the atmosphere, solid substances from the lithosphere, and ionic and nonionic substances from the oceans. Alternative standard exergy reference environments that have gained acceptance for engineering evaluations are detailed by [1] and [16].

The use of standard chemical exergies greatly facilitates the application of exergy principles. The term *standard* is somewhat misleading, however, for no one specification of the environment suffices for all applications. Still, chemical exergies calculated relative to alternative specifications of the environment generally agree well. For a broad range of engineering application, the simplicity and ease of use of standard chemical exergies generally outweighs any slight lack of accuracy that might result.

A common feature of standard exergy reference environments is a gas phase intended to represent air, that includes N_2 , O_2 , CO_2 , $H_2O(g)$, and other gases. Each

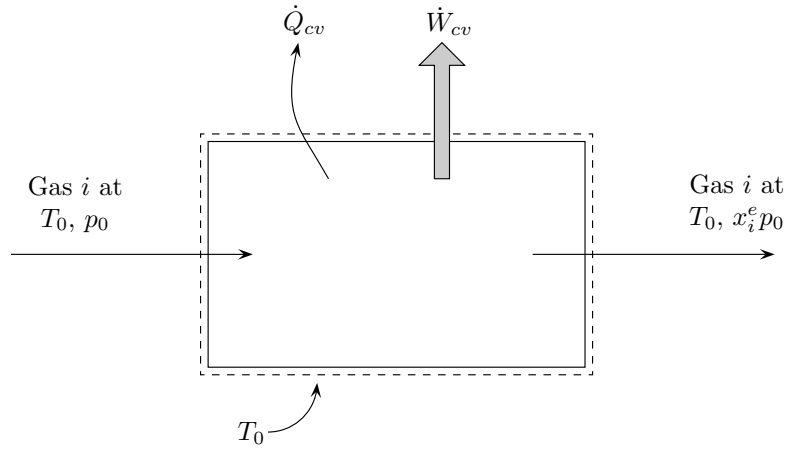


Figure A.3. Device for evaluating the chemical exergy of a gas.

gas i present in the gas phase is at temperature T_0 and partial pressure $p_i^e = x_i^e p_0$, where the superscript e is the mole fraction of gas i in the gas phase. Referring to the device at steady state shown in A.3, the standard chemical exergy for a gas included in the environmental gas phase can be evaluated as follows: Gas i enters at temperature T_0 and pressure p_0 , expands isothermally with heat transfer only with the environment, and exists to the environment at temperature T_0 and partial pressure p_i^e . The maximum theoretical work per mole of gas i would be developed when the expansion occurs without internal irreversibilities. Accordingly, with energy and entropy balances together with the ideal gas equation of state, the chemical exergy per mole of i is

$$\begin{aligned}\bar{e}_i^{CH} &= -\bar{R}T_0 \ln \frac{x_i^e p_0}{p_0} \\ &= -\bar{R}T_0 \ln x_i^e\end{aligned}\tag{A.17}$$

The chemical exergy of a mixture of n gases, each of which is present in the environmental gas phase, can be obtained similarly. We may think of a set on n devices such as shown in A.3, one for the gas in the mixture. Gas i , whose mole fraction in the gas mixture at T_0, p_0 is x_i , enters at T_0 and the partial pressure $x_i p_0$. As before, the gas exists to the environment at T_0 and the partial pressure $x_i^e p_0$. Paralleling the previous development, the work per mole of i is $-\bar{R}T_0 \ln(x_i^e/x_i)$. Summing over all components, the chemical exergy per mole of mixture is

$$\bar{e}_{mixture}^{CH} = -\bar{R}T_0 \sum_i x_i \ln \frac{x_i^e}{x_i}.$$

This expression can be written alternatively with equation (A.17) as

$$\bar{e}_{mixture}^{CH} = \sum_i x_i \bar{e}_i^{CH} + \bar{R}T_0 \sum_i x_i \ln x_i.\tag{A.18}$$

Equation (A.18) remains valid for mixtures containing gases, for example, gaseous fuels, other than those assumed present in the reference environment and can be extended to mixtures that not adhere to the ideal gas model.

A.3 Exergy Balance

As for extensive properties mass, energy, and entropy, exergy balances can be written in alternative forms suitable for particular applications of practical interest.

The change in total exergy ΔE_{sys} of a closed system caused through transfers of energy by work and heat between the system and its surroundings is given by

$$\Delta E_{sys} = E_{sys,2} - E_{sys,1} = E_q - E_w - I \quad (A.19)$$

The exergy transfer E_q associated with heat transfer Q is

$$E_q = \int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q$$

where T_b is the temperature at which heat transfer crosses the system boundary. The exergy transfer E_w associated with the transfer of energy by work W is given by

$$E_w = W + p_0(V_2 - V_1)$$

The last term I accounts for the time rate of exergy destruction due to irreversibilities within the system and is related to the rate of entropy generation within the system by

$$I = T_0 \dot{S}_g$$

The counterpart of equation (A.19) applicable to control volumes include additional terms that account for exergy transfers into or out of a control volume where streams of mass enter and exits:

$$\begin{aligned} \frac{dE_{sys}}{dt} = & \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \left(\dot{W}_{cv} - p_0 \frac{dV}{dt}\right) \\ & + \sum_i \dot{m}_i e_i - \sum_e \dot{m}_e e_e - \dot{I}. \end{aligned} \quad (A.20)$$

In equation (A.20), the term dE_{sys}/dt represents the time rate of change of the exergy on the control volume. As in the control volume energy rate balance, \dot{W}_{cv} represents the time rate of energy transfer by work other than flow work. The term $\dot{m}_i e_i$ accounts for the time rate of exergy transfer accompanying mass flow and flow work at inlet i . Similarly, $\dot{m}_e e_e$ accounts for the time rate of exergy transfer accompanying mass flow and flow work at exit e . The exergy transfer rate at control volumes inlets and exits are denoted, respectively, as $\dot{E}_i = \dot{m}_i e_i$ and $\dot{E}_e = \dot{m}_e e_e$. Finally, \dot{I} accounts for the time rate of exergy destruction due to irreversibilities within the control volume.

At steady state, equation (A.20) reduces to:

$$\sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv} + \sum_i \dot{m}_i e_i - \sum_e \dot{m}_e e_e = \dot{I}. \quad (A.21)$$

To complete the introduction of the control volume exergy balance, an equation for evaluating the terms $\dot{m}_i e_i$ and $\dot{m}_e e_e$ appearing in equations (A.20) and (A.21) is considered next: When mass enters or exits a control volume, exergy transfers accompanying mass flow and flow work occur, that is

$$\left[\begin{array}{c} \text{time rate of exergy transfer} \\ \text{accompanying mass flow} \end{array} \right] = \dot{m} e_{sys} \quad (\text{A.22})$$

On time-rate basis the flow work is $\dot{m}(p v)$, and the accompanying exergy transfer is given by

$$\left[\begin{array}{c} \text{time rate of exergy transfer} \\ \text{accompanying flow work} \end{array} \right] = \dot{m}(p v - p_0 v) \quad (\text{A.23})$$

The sum of both contributions is the time rate of exergy transfer accompanying mass flow and flow work, symbolized by \dot{E} and it is also called flow exergy.

$$\dot{E} = \dot{m}[e_{sys} + (p v - p_0 v)] \quad (\text{A.24})$$

The sum in square brackets on the right is the specific flow exergy, that is,

$$e = e_{sys} + (p v - p_0 v) \quad (\text{A.25})$$

Introducing equations (A.13) and (A.15), in equation (A.25) the equation for specific flow exergy is obtained

$$e = \underline{h - h_0 - T_0(s - s_0)} + \frac{V^2}{2} + gz + e^{CH}, \quad (\text{A.26})$$

where h_0 and s_0 denote the specific enthalpy and entropy, respectively, at the restricted dead state. The underlined term can be identified as the physical component of the exergy of a flow stream, written as

$$e^{PH} = h - h_0 - T_0(s - s_0). \quad (\text{A.27})$$

Review of Lineal Algebra

In this appendix we will review some aspects of linear algebra that been used in the book. We will take special attention in notation aspects. Some numerical methods for solving linear system of equation are introduced here.

B.1 Set Theory

A *set* is a well-defined collection of things. By well defined we mean that given any object it is possible to determine whether or not it belong to the set. For example, the set $S = \{x|x \geq 0\}$ defines the set of all nonnegative numbers. $x = 2$ is an element of the set S and is written as $2 \in S$.

The *union* of two sets P and Q defines another set R such that

$$R = P \cup Q = \{x \in P \text{ or } x \in Q\}.$$

The *intersection* of two set P and Q defines another set R such that

$$R = P \cap Q = \{x \in P \text{ and } x \in Q\}.$$

P is a *subset* of Q , written $P \subset Q$, if every element of P is in Q . The *empty set*, denoted by \emptyset , is a set with no elements in it.

Disjoints sets have no elements in common, if P and Q are disjoints sets, then $P \cap Q = \emptyset$.

B.2 Vectors

A vector is an ordered set of real numbers. For instance, $\mathbf{a} = (a_1, a_2, \dots, a_n)$ is a vector of n elements or components. Let two vectors $\mathbf{a} = \{a_1, a_2, \dots, a_n\}$ and $\mathbf{b} = \{b_1, b_2, \dots, b_n\}$, then the sum is defined as:

$$\mathbf{a} + \mathbf{b} = (a_1 + b_1, \dots, a_n + b_n)$$

The scalar product for any real number α is:

$$\alpha \mathbf{a} = (\alpha a_1, \dots, \alpha a_n)$$

The vector $\mathbf{0} = (0, 0, \dots, 0)$ is called null vector. The vector $\mathbf{u} = (1, 1, \dots, 1)$ is called unity vector.

The inner product of two vector, written $\mathbf{a} \cdot \mathbf{b}$, is a real number given by:

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^n a_i b_i$$

B.3 Matrices

A matrix \mathbf{A} of size $n \times m$ is a rectangular array (table) of numbers with m rows and n columns, as for example:

$$\mathbf{A}_{(2 \times 3)} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

In this example \mathbf{A} is matrix of two rows and three columns. In general an $n \times m$ matrix is written as

$$\mathbf{A}_{(n \times m)} = [a_{ij}]_{i=1, \dots, n}^{j=1, \dots, m}$$

each element a_{ij} of the matrix, is the element in the i -th row and j -th column of \mathbf{A}

Here continue the appendix

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